# Assessment and status of Swordfish in the SW Indian Ocean (1950-2008) WP

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## Summary

During the WPB, a statistical catch-age model (SCAM) was fit to catch and CPUE data in the SW region of the Indian Ocean. This model was not fit to catch-at-age or catch-at-length (composition data) due to time constraints; therefore, age-selectivity was assumed and not estimated internally. A logistic function was used to model age-specific selectivity with the 50% vulnerability fixed at age 3, and a standard deviation of 0.75 years.

The following assumptions were made:
1. M is equal to 1.5 times the von Bertalanffy growth coefficient and is age-independent and time invariant.
2. Growth and weight at age is constant.
3. All CPUE indicies are proportional to abundance with lognormal errors and a coefficient of variation arbitrarily set at 0.1.
4. Recruitment follows a Beverton-Holt form with an informative prior on steepness, Beta(alpha=8,beta=5).
5. Age selectivity is logistic with a mean age of 3 years and a standard deviation of 0.75 years.

Two alternative scenarios for the SW Indian Ocean were presented, 1) CPUE indices from JPN LL, TWN LL and the La Reunion fishery were all fit, with the exception of the 2008 index for the TWN LL fishery. In the second scenario, the entire TWN LL CPUE index was removed from the fitting. Phase plots of spawning stock biomass relative to SBmsy and fishing mortality relative to Fmsy (KOBE plots) are shown in Figure i.

Scenario 1 (Fig. ia) resulted in a median estimate for MSY 6,258 mt (and 90% credible interval, or CI of 5,455 mt and 7,138 mt), with corresponding estimates of Fmsy and SBmsy of 0.367 (0.29 and 0.45 CI) and 4,717 mt (3,000 mt and 7,277 mt CI), respectively (note these estimates result in an extremely productive stock with a steepness value of 0.978). Median estimates of spawning biomass depletion (Sbt/Sbo) were 3.2% with a 90% CI of 1.8% to 5.5%.

Scenario 2 (Fig. ib) resulted in a similar median estimate for MSY of 6,357 mt (5,500 mt and 7,266 mt CI) as that of Scenario 1. Median estimates of Fmsy were 0.39 (0.31 and 0.48 CI) and median estimates of SBmsy were 4,160 mt (2,667 mt and 6,292 mt CI). Estimates of the steepness parameter for Scenario 2 is also similar to Scenario 1 at 0.984. Estimates of spawning biomass depletion were 8.0% (5.34% to 11.35% CI).

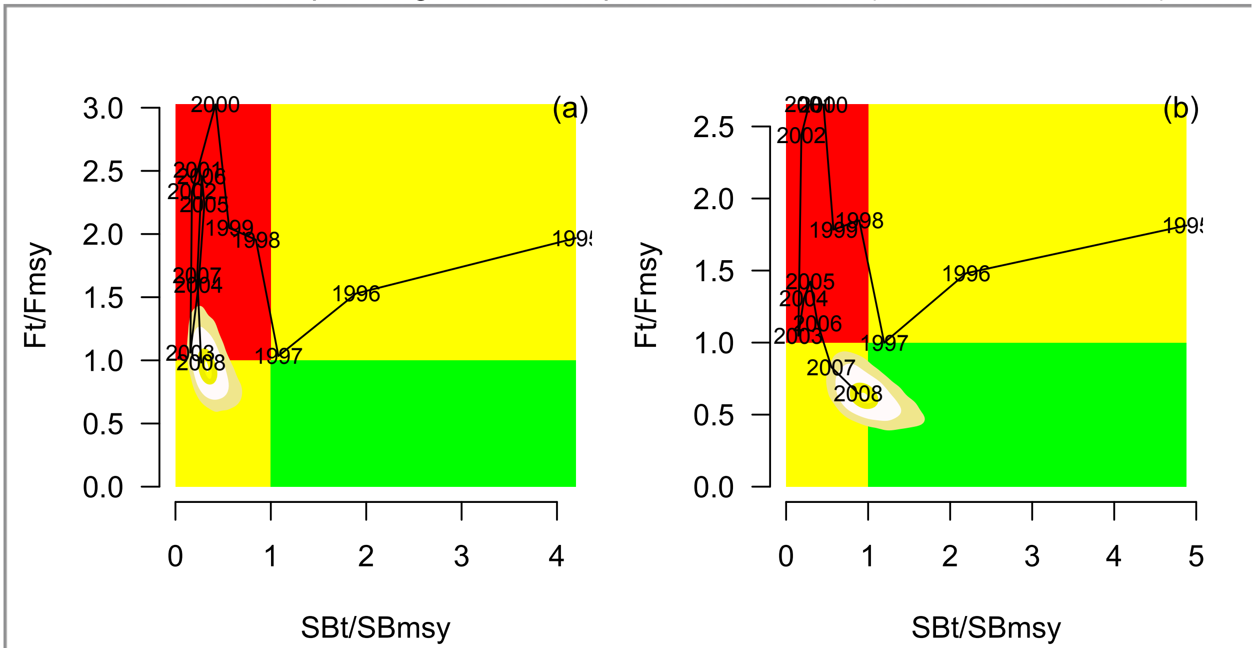


Figure i. Phase plots illustrating the results of Scenario 1 (a) and Scenario 2 (b) of the SCAM model for swordfish in the SW region of the Indian Ocean. The fried egg represents uncertainty (obtained from posterior integration using a Metropolis Hastings algorithm), where each color change spans the 5th, 25th, 75th and 95th percentiles.

Future projections of spawning stock biomass and fishing mortality rate scenarios are based on 5 catch levels ranging from -20%, -10%, 0, 10% and 20% changes in the 2008 catch level (Table i). Probabilities of exceeding MSY based reference points are based on posterior samples and future recruitment variability based on a CV of 0.4.

Table i. Strategy Matrix for setting management measures based on two Scenarios. The management targets are assumed to be MSY based; the probability of exceeding Fmsy and reducing the spawning stock below SBmsy for 3 alternative catch options ranging from 5,140 mt to 7,711 mt in 3 and 10 years are shown.

| **Management Target** | **Time frame** | | **Probability of Meeting Target given catch option** | | |
|---|---|---|---|---|---|
| | | | **5,140 mt** | **6,426 mt** | **7,711 mt** |
| P(Ft>Fmsy) | In 3 years (2011) | S1 | <0.01 | 0.494 | 0.242 |
| | | S2 | <0.01 | 0.031 | 0.206 |

| Management Target | Time frame | | Probability of Meeting Target given catch option | | |
|---|---|---|---|---|---|
| | | | **5,140 mt** | **6,426 mt** | **7,711 mt** |
| | In 10 years (2018) | S1 | 0.025 | 0.145 | 0.522 |
| | | S2 | 0.016 | 0.110 | 0.481 |

| Management Target | Time frame | | Probability of Meeting Target given catch option | | |
|---|---|---|---|---|---|
| | | | **5,140 mt** | **6,426 mt** | **7,711 mt** |
| P(SBt<SBmsy) | In 3 years (2011) | S1 | 0.107 | 0.200 | 0.343 |
| | | S2 | <0.01 | <0.01 | <0.01 |
| | In 10 years (2018) | S1 | 0.031 | 0.132 | 0.422 |
| | | S2 | 0.016 | 0.092 | 0.387 |

## Introduction

The 2010 Working Party on Billfish, held in the Seychelles in July 2010, noted that there has been a rather strong decline the in the Japanese LL CPUE data for swordfish in the early 1990s. There have also been substantial declines in the CPUE for the La Reunion LL fishery that operates exclusively in the southwest region of the Indian Ocean. More recently the CPUE index from the Taiwanese LL fishery also shows drastic declines in CPUE. Participants of the 2010 expressed strong concerns about local depletion of swordfish stocks in this region.

In this working paper, I use a Statistical Catch-Age Model (SCAM) to reconstruct swordfish abundance and estimate MSY based reference points by jointly fitting the model to CPUE data from the Japanese Longline fishery (JPN LL), the Taiwanese LL (TWN LL) and La Reunion Longline (REU LL) fishery and total aggregate catch from 5 separate fishing fleets that operate in this region.

## Methods

Estimates of historical swordfish abundance in the SW region of the Indian Ocean was reconstructed using a Statistical Catch-Age Model (SCAM). A full description of the model equations are provided in Appendix A. Historical catch between 1950 and 2008 for the SW region were provided at the WPB meeting, as well as, catch per effort (CPUE) data from the JPN LL, TWN LL and REU LL fisheries. These data can be found in Working papers (XXX) on the IOTC website. No composition information were used in the fitting of SCAM, so selectivity for the fishery was assumed to follow a logistic distribution with a mean age of 3 years and a standard deviation of 0.75 years. These selectivity parameters were chosen by comparing the predicted length frequency distribution from SCAM to the observed length frequencies collected in 2008. Although the selectivity parameters could be jointly estimated, the were arbitrarily set due to time constraints. Recruitment follows a Beverton-Holt form with an informative prior on steepness, Beta(alpha=8,beta=5).

Information on swordfish growth was obtained from a recent analysis on length-age data in the northern areas of the Indian Ocean (Sheng-Ping Wang, Pers. Comm. National Taiwan Ocean University, Keelung 202 Taiwan IOTC-2010-WPB-08). The average asymptotic lengths and growth coefficients for males and females were average to derived a growth curve for males and female combined. I used life-history invariance methods (Jensen 1996) to derive assumed values for natural mortality and maturity-at-age (see Table 1 for derived values). Maturity-at-age was assumed to follow a logistic function and fecundity-at-age is assumed to be proportional to body weight of sexually mature fish. All age-schedule information is shown graphically in Figure 1.

Table 1. Assumed parameter values for the SCAM model.

| Parameter | Value | Source & Description |
|---|---|---|
| Linf | 254 | Wang, pers comm. Asymptotic length |
| k | 0.1535 | Wang, pers comm. Metabolic growth coefficient |
| to | -2.0895 | Wang, pers comm. age-at-zero length |
| a | 9.133 x10-6 | Wang, pers comm. allometric length-weight scaler |
| b | 3.012 | Wang, pers comm. allometric length-weight power |
| A | 25 | Arbitrary, plus group age. |
| M | 0.23 | Natural mortality (M=1.5*k, Jensen 1996). |
| ah | 7.15 | Age at 50% maturity (log(3)/k, Jensen 1996). |
| gh | 0.715 | Std at age at maturity in logistic function. |
| am | 3 | Arbitrary, age-50% vulnerability |
| gm | 0.75 | Arbitrary, standard deviation in logistic selectivity |

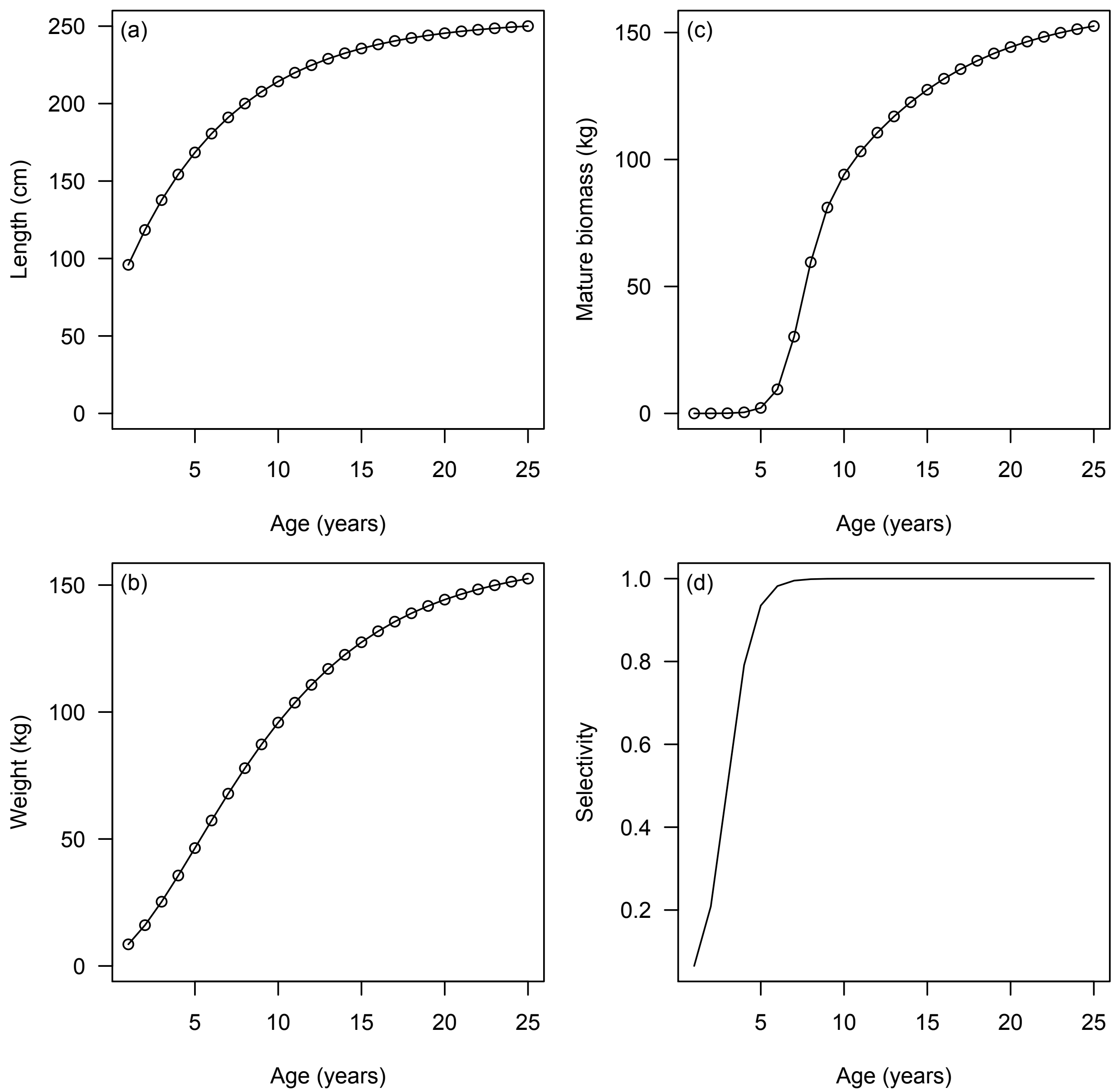


Figure 1. Age-schedule information used in the SCAM assessment, (a) mean length-at-age, (b) mean weight-at-age, (c) age-specific spawning biomass, and (d) assumed age-specific selectivity curve.

The assessment model was fit to time series data on total catch in the SW region, and CPUE indices from the JPN LL, REU LL and TWN LL fishing fleets (Figure 2). Two separate scenarios were explored in this assessment. Scenario 1 uses all the available information with exception of the 2008 CPUE index from the TWN LL fishing fleet (note this value is not shown in Figure 2). In Scenario 2, the TWN LL CPUE index was omitted. All three indexes show declining trends in abundance between 1994-5 to 2000-3; however, in recent years there is a clear contradiction in trends between the TWN and JPN indexes. There was no real rational for discarding the TWN index over the JPN index in these two scenarios, rather the second scenario was included just to demonstrate the relative impacts of omitting the two series on estimates of reference points. In the assessment, residual errors in the catch was assumed to have a standard deviation of 0.05, and all the CPUE indices were assumed to have a CV of 0.1.

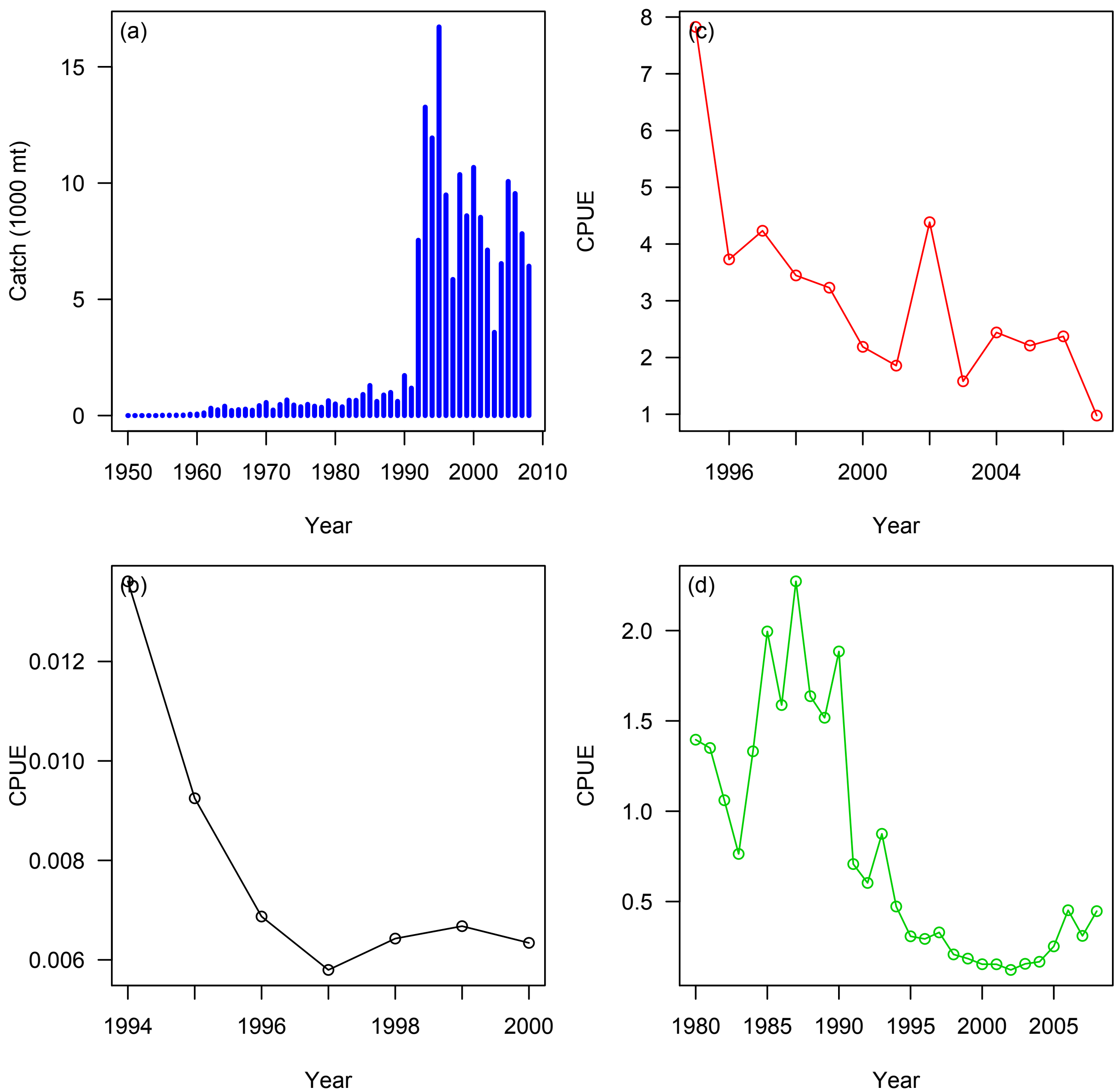


Figure 2. Data used in the SCAM assessment. The catch of swordfish from the SW region is shown in panel (a), the REU LL CPUE in panel (b), the TWN LL CPUE index (excluding 2008) in (c), and the JPN LL CPUE index in panel (d).

Parameters were estimated using the non-linear search routine that is built into AD Model Builder, and uncertainty was estimated by numerically integrating the joint posterior distribution using the built in Metropolis-Hastings algorithm (ADMB Project 2009). Estimated reference points were based on Maximum Sustainable Yield (MSY) and were calculated based on steady state conditions,a Beverton-Holt recruitment model (Martell et. al. 2008), and estimated model parameters.

Stochastic stock projections 10 years into the future based on fixed catch values was carried out by using the posterior samples and drawing future recruitment deviations from a lognormal distribution with a coefficient of 0.4. Probabilities of exceeding Fmsy and the spawning stock falling below SBmsy were computed for catch options ranging from -20%, -10%, 0, 10%, 20% over the status quo catch of 6,426 t that was taken in 2008.

**Results**

*Scenario 1.*

Current estimates of spawning stock biomass are below estimated levels of SBmsy and current estimates of fishing mortality are at or near estimates of Fmsy (Figure 3). There is no relative abundance information between 1950 and 1980, so estimates of abundance during this period are largely determined by the mean recruitment levels that are determined from the trend data post 1980. The longest CPUE series is the JPN LL series from 1980 to 2008 and the assessment model does a fairly decent job fitting the longer term trends in the JPN data (Figure 4). Between 1980 and 1985 there is an increase in the JPN LL data; absent any age-composition data during this period, this increase is explained by positive recruitment. Between 1991 and 1992, there is a large decline in the JPN LL cpue data; the assessment model interprets this change in abundance as a sudden increase in fishing mortality rates after 1991 and a period of poor recruitment starting in the mid 1980s and lasting until the mid 1990s.

Estimates of steepness for the stock recruitment relationship were extremely high (0.978). Estimated reference points for scenario 1 are summarized along with scenario 2 in Table 2. Of concern is the estimates of SBmsy that are less than the estimate of the maximum sustainable yield. Given only the trend information in the CPUE series and the catch history, the data suggest this is an extremely productive stock that can sustain incredibly high fishing mortality rates.

Table 2. Estimates of reference points and depletion levels. Note the dome selectivity scenario is based on the same data used in Scenario 2.

| Statistic | Scenario 1 | Scenario 2 | Dome-shaped Selectivity |
|---|---|---|---|
| MSY | 6409 mt | 6731 mt | 6879 mt |
| Fmsy | 0.377 | 0.403 | 0.35 |
| SBmsy | 4,459 mt | 4,209 mt | 4984 |
| SBt2008/SBo | 0.0241 | 0.0707 | 0.054 |

| Statistic | Scenario 1 | Scenario 2 | Dome-shaped Selectivity |
|---|---|---|---|
| SBt2008/SBmsy | 0.269 | 0.880 | 0.616 |
| Ft2008/Fmsy | 0.982 | 0.646 | 0.663 |

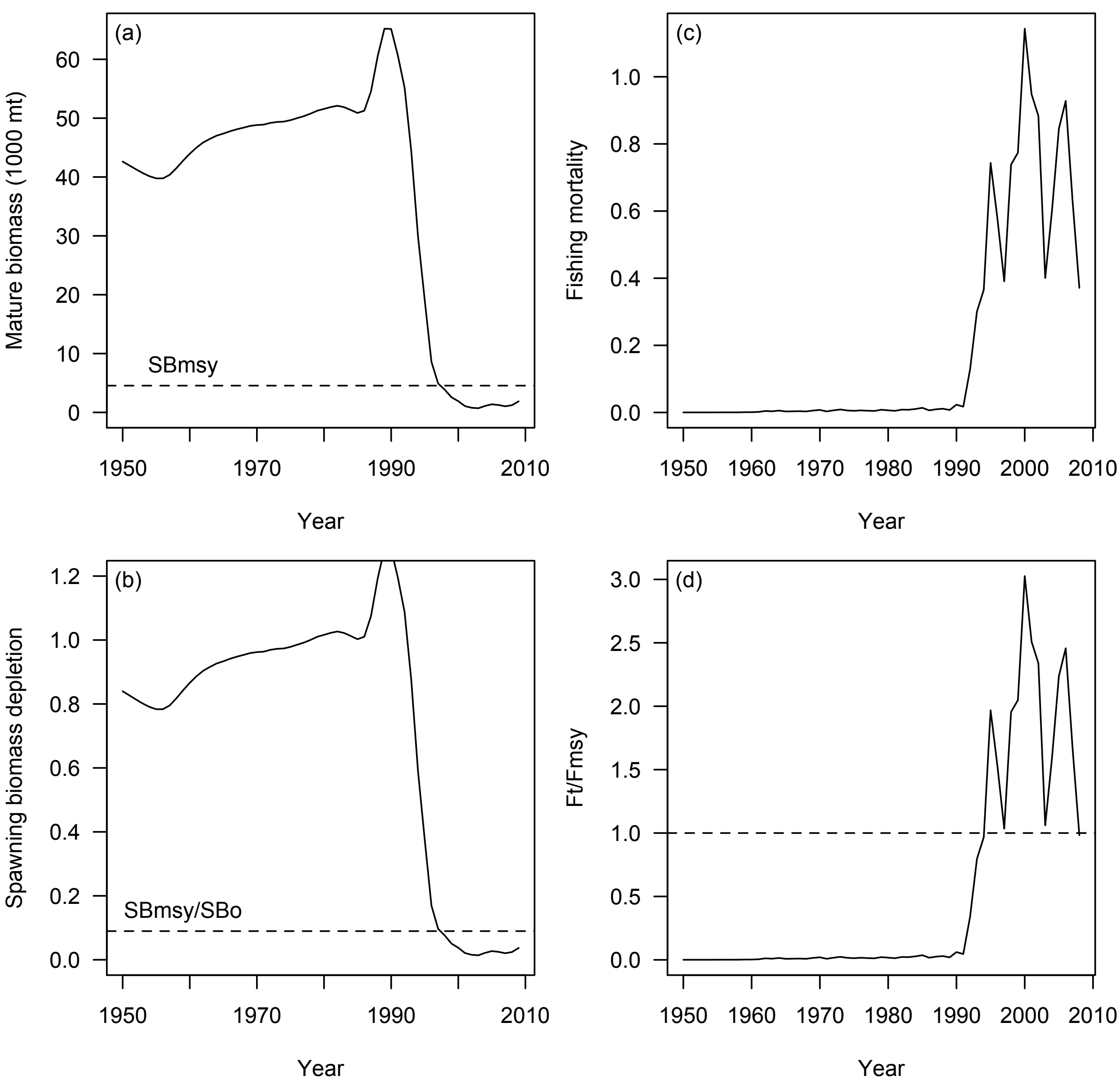


Figure 3. Estimates of mature biomass (surrogate for female spawning stock biomass) from 1950 to 2009 based on scenario 1; (a) relative to unfished biomass (b), and estimates of fishing mortality (c) relative to estimates of Fmsy (d).

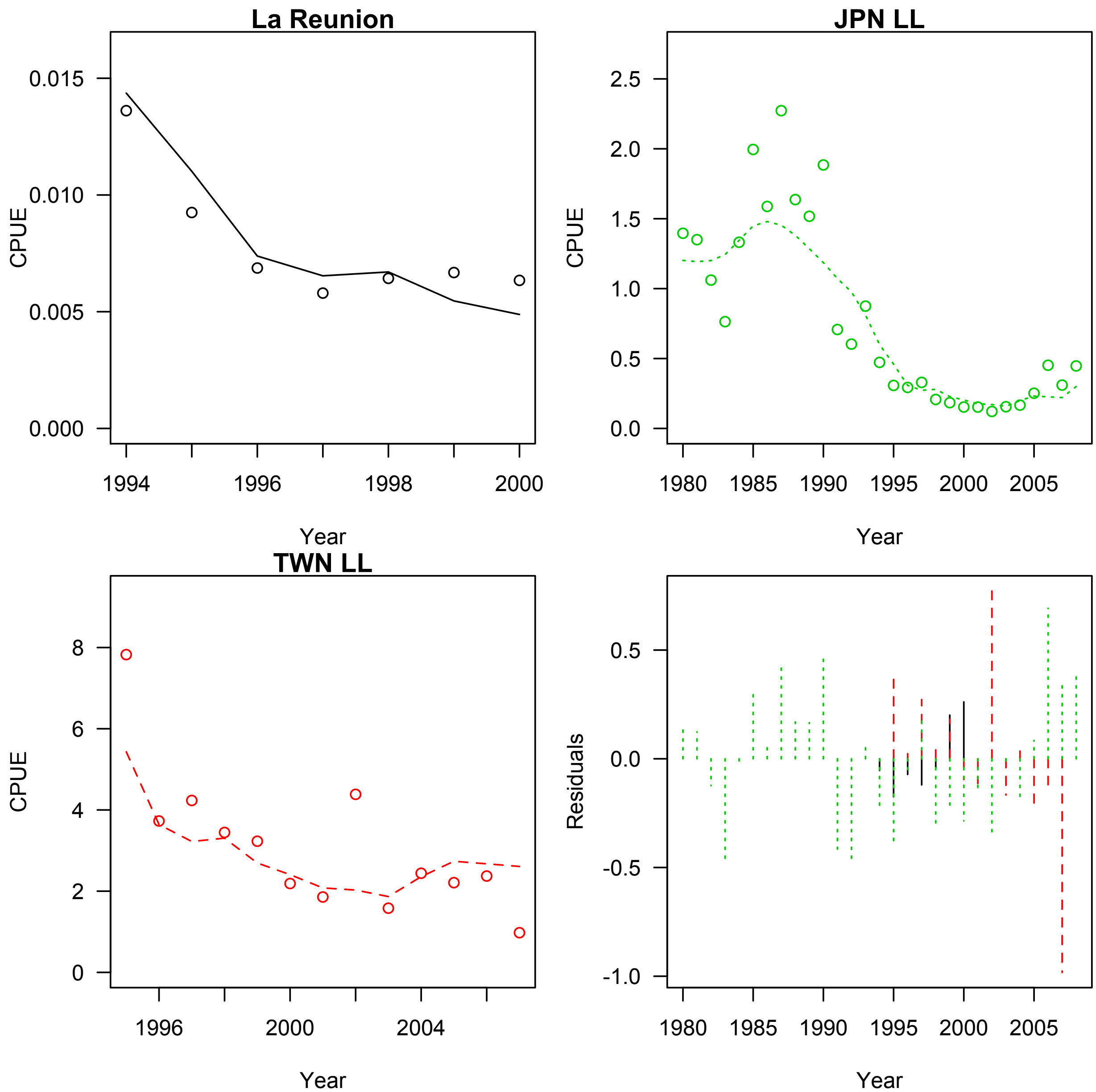


Figure 4. Fits to the CPUE data (Scenario 1) from La Reunion, Taiwan LL, and the Japan LL data, and the corresponding residuals from all series in the lower right hand panel.

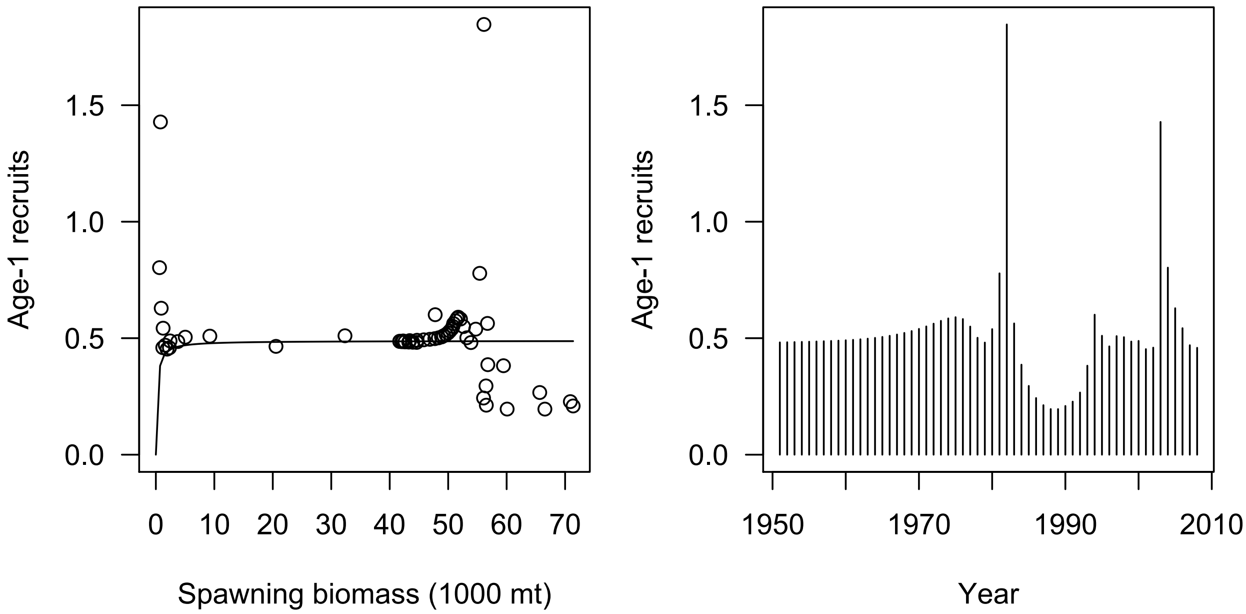


Figure 5. Stock-recruitment relationship for swordfish in the SW region and estimates of age-1 recruits.

*Scenario 2*

In scenario 2, the TWN LL CPUE data was omitted from the statistical fitting criterion, and estimates of spawning stock biomass are still extremely low relative to the unfished levels (Table 2, Figure 6). Estimated reference points are fairly similar between scenario 1 and scenario 2 (Table 2). The main difference between the two scenarios is the recent increase in abundance that is associated with the apparent increase in the JPN LL CPUE since 2003. There is very good correspondence between the JPN LL CPUE and the estimated vulnerable biomass since the mid 1990s, and also reasonable correspondence in the La Reunion CPUE (Figure 7). In the case of the TWN LL CPUE, there is general agreement with the declining trends between 1996 and 2003, but after 2003 the estimated abundance diverges from the rapidly declining trends in the observed CPUE index.

Apparent trends in age-1 recruits in Scenario 2 are very similar to that of scenario 1 (Figure 8), with an increase in age-1 recruits in the early 1980s that is required to explain the apparent increase in JPN LL CPUE between early 1980s and late 1980s. In the late 1980s and early 1990s there age-1 recruitment is below average and partially explains the rapid decline in the JPN LL CPUE between 1991 and 1992. At the same time, fishing mortality rates suddenly increase corresponding to the large increase in reported catches in 1992.

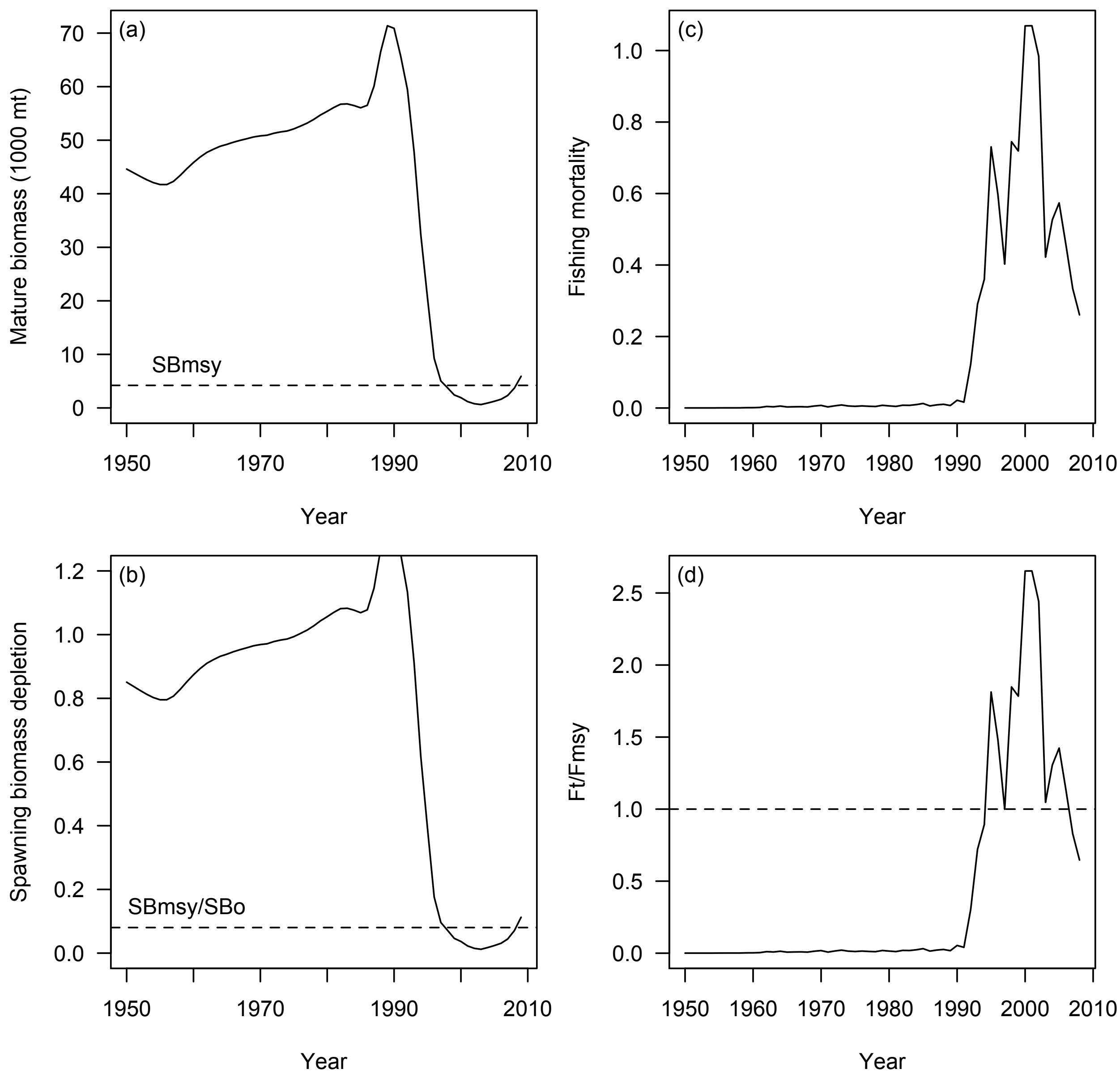


Figure 6. Estimates of mature biomass (surrogate for female spawning stock biomass) from 1950 to 2009 based on scenario ; (a) relative to unfished biomass (b), and estimates of fishing mortality (c) relative to estimates of Fmsy (d).

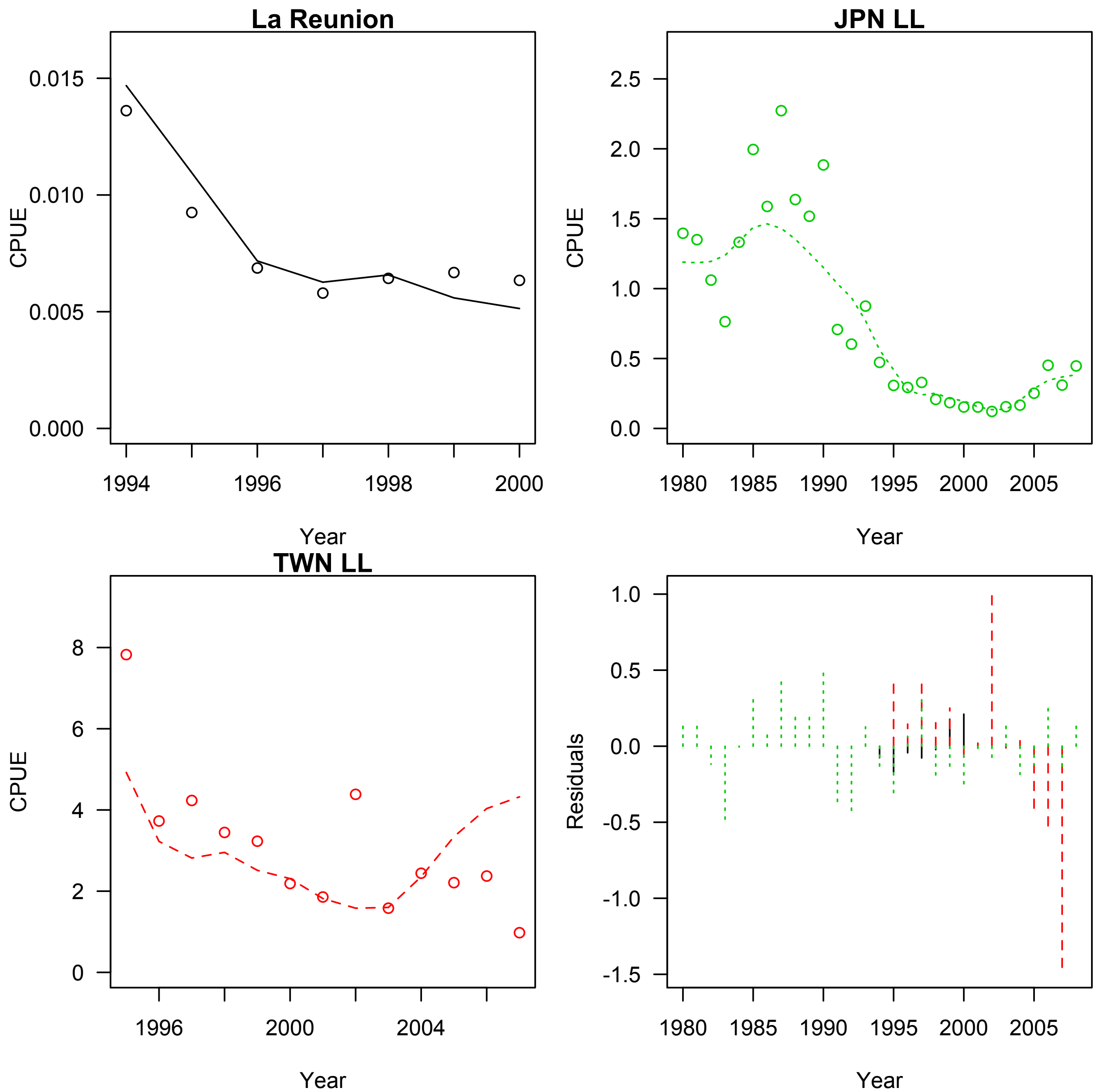


Figure 7. Observed (points) and predicted (lines) CPUE from scenario 2. THe model was fit to the CPUE data from La Reunion and the Japan LL. The model was not fit to the TWN LL data, but is shown for comparing trends.

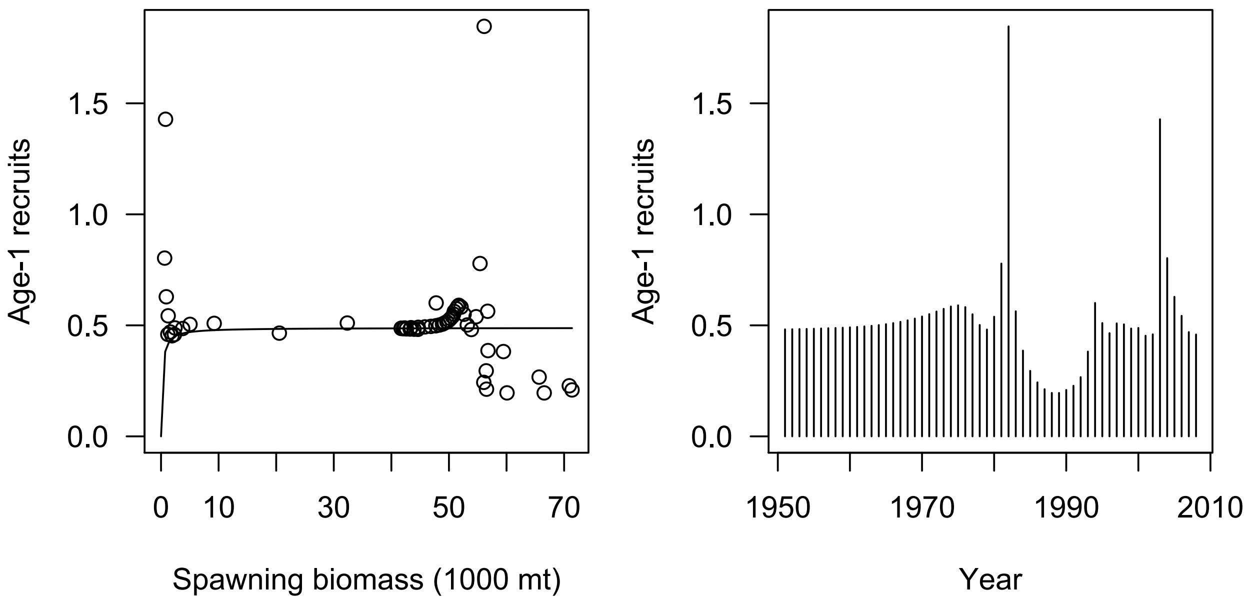


Figure 8. Estimated age-1 recruits versus spawning stock biomass (left panel) and estimates of age-1 recruits (right panel) for Scenario 2.

*Stock Status & Projections*

For both scenarios 1 and 2, the current maximum likelihood estimates of stock status are that the stock is overfished (i.e., below SBmsy levels) and that recently fishing mortality rates have fallen below estimates of Fmsy as catches have fallen from 10,069 t in 2005 to 6,426 t in 2008 (Figure 9). Uncertainty in the current estimates of stock status in 2008 was characterized by taking 2500 systematic samples from the joint posterior distribution and plotting the corresponding spawning biomass and fishing mortality rates as a contour plot (that resembles a fried egg in Figure 9). In Scenario 1, roughly half of the fried egg is in the overfished--overfishing zone, and the other half is in the overfished zone. In other words, there is nearly a 50% chance that the stock is overfished and overfishing is occurring under scenario 1. In the case of scenario 2, there is a greater than 50% chance the stock is overfished, but no overfishing in 2008.

Deterministic projections of spawning stock biomass 40 years in to the future suggest that a 10% decrease in catch over the current 2008 harvest (~5,100 mt) would prevent further declines in spawning stock biomass for both scenarios (Figure 10.).

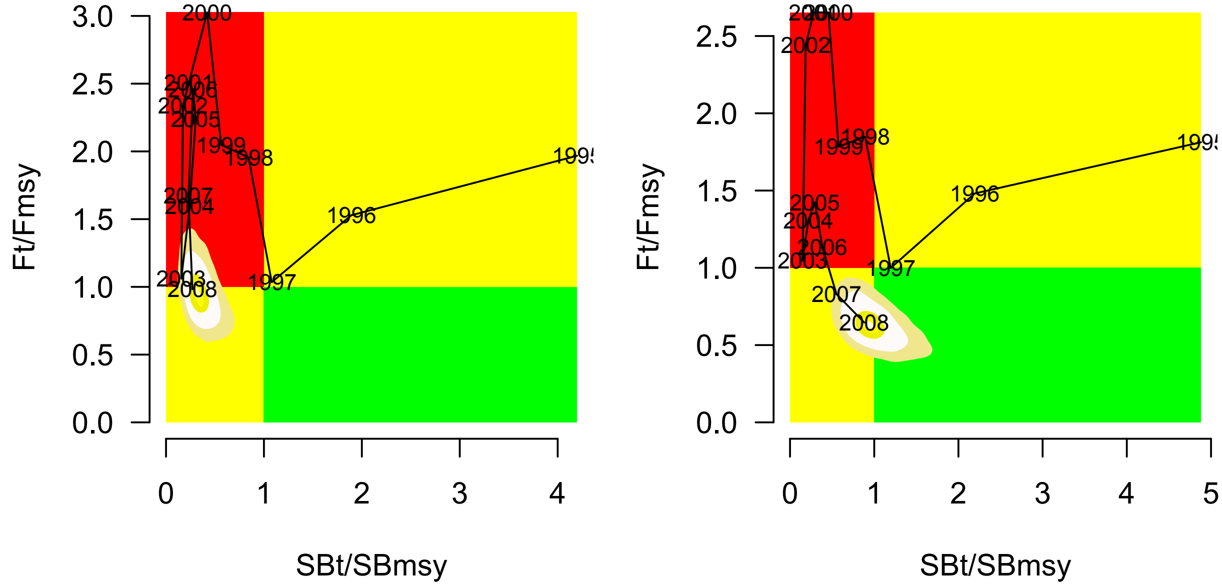


Figure 9. Phase plots of estimated spawning stock biomass relative to the estimate of SBmsy versus fishing mortality rates relative to the estimate of Fmsy for Scenario 1 and 2 (left and right panels, respectively). The fried egg represents uncertainty where each color change spans the 5th, 25th, 75th and 95th percentiles.

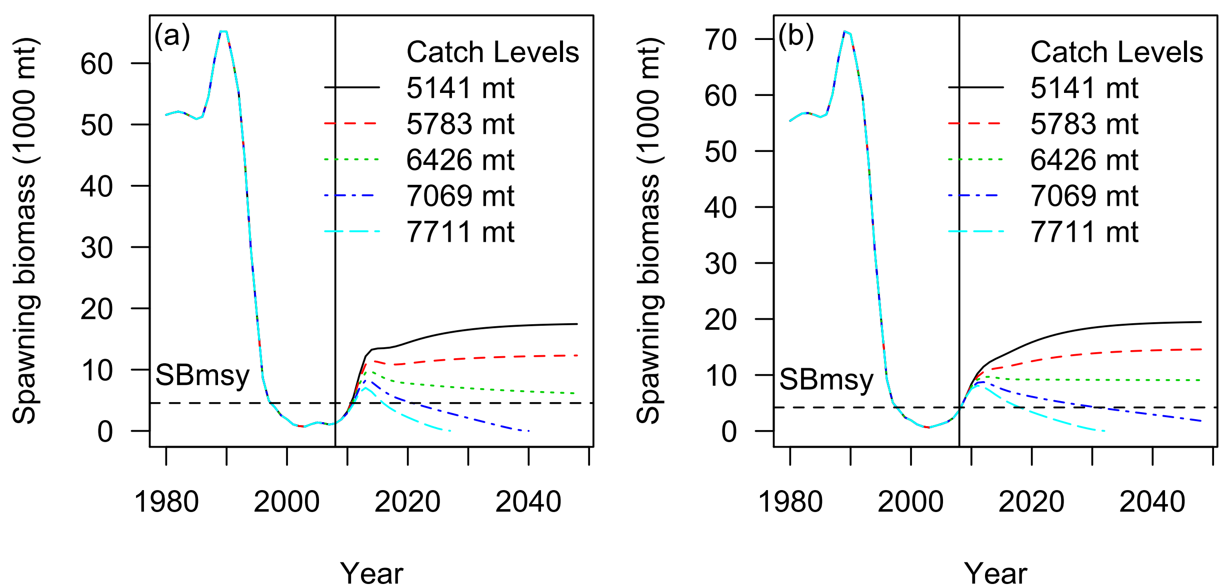


Figure 10. Deterministic projections of spawning stock biomass 40 years into the future under constant catch levels ranging from 5,141 mt to 7,711 mt for Scenario 1 (a) and Scenario 2 (b).

## Discussion

Points:
1) Summary of results
2) Assumptions, data, model,
3) Weighting of scenarios
4) Realism, hard to believe that MSY levels are greater than estimates of SBmsy for Swordfish.

There are strong indications that swordfish abundance in the SW Indian ocean region have undergone rather severe declines at least since the 1990s. All three relative abundance indices developed for this region have strong declines in abundance throughout the 1990s. The Reunion index has declined to 45% of its maximum level between 1994 and 2000. Ignoring the 2008 CPUE index, the TWN LL index has declined to 12% of its maximum level between 1995 and 2007. Finally the JPN LL index peaked in 1987, declined to a low of 5.3% of its maximum in 2002, and has subsequently increased to 19.6% of its maximum in 2008. Assuming that these CPUE indices are proportional to abundance implies a very rapid decline in abundance in just a few short years. This decline cannot be explained by increases in fishing mortality rates alone as estimated catches increased rapidly between 1991 and 1992, but also stayed at these high levels for nearly 2 decades now.

The assessment model used herein (SCAM) makes a number of assumptions that are critical in determining current stock status and estimates of stock productivity. First, the instantaneous natural mortality rate was assumed at 0.23. Increase the assumed value of M (or the von Bertalanffy growth coefficient) results in increased estimates of stock productivity and increases in the values of MSY and Fmsy reference points. Selectivity was also assumed to be a logistic function of age, where the age-at-50% vulnerability was arbitrarily assumed at 3 years. Increasing the age-at-50% vulnerability to 5 years results in much more optimistic estimates of current stock status, and vice versa. Also, estimates of SBmsy increase relative to estimates of MSY as the fishery targets larger older fish. Assuming dome-shaped selectivity was also partially explored and only the estimated reference points and depletion levels were presented. Under the assumption of dome shaped selectivity, estimates of stock status lie in between the results obtained for scenarios 1 and 2.

Alternative assumptions about data weighting and how much variation there is in the annual recruitment deviations will also have a large influence on model estimates. Under the two scenarios presented here, it was assumed that the standard deviation in all CPUE data were 0.1 and the standard deviation in recruitment deviations was 0.4 (i.e., a mixed error model). If a deterministic stock-recruitment relationship was assumed, the resulting estimates of spawning stock biomass depletion would be very similar; however, estimates of stock productivity and scale would have to increase substantially in order to explain the observed declining trends in the CPUE data. Therefore, it is probably more conservative to arbitrarily assume some level of recruitment variation rather than a deterministic, or observation error only model.

There was limited time to explore or discuss alternative weighting schemes for the CPUE indices using in estimating stock status and reference points for the SW Indian Ocean.  Further examinations of the raw data that was used to reconstruct the CPUE indices should be explored.  The selective use of only certain areas has the potential to strongly bias the CPUE index (e.g., Walters 2003), this should be explored in much greater detail.

Finally, its hard to conceive that the estimated spawning biomass at MSY levels (SBmsy) is actually less than the estimated maximum sustainable yield for swordfish in the SW Indian ocean.  This is inconsistent with estimates from other regions, or estimates obtained when data for each of the regions in the Indian ocean is pooled into a single region.  Reducing estimates of apparent swordfish productivity in the SW region would either have to be explained by a hyperdepleted CPUE index for this region, or large annual immigration events are occurring.

## References

Jensen, A. (1996). Beverton and holt life history invariants result from optimal trade-off of reproduction and survival. Can. J. Fish. Aquat. Sci., 53:820–822.

Martell, S. J. D., Pine, W. E., and Walters, C. J. (2008). Parameterizing age-structured models from a fisheries management perspective. Can. J. Fish. Aquat. Sci., 65:1586–1600.

Walters, C. (2003). Folly and fantasy in the analysis of spatial catch rate data. Canadian Journal of Fisheries and Aquatic Sciences, 60(12):1433–1436.

Appendix A. Description of the Statistical Catch Age Model (SCAM).

The statistical catch-at-age model (SCAM) is based on the general framework of Fournier and Archibald (1982). SCAM is coded in AD Model Builder (ADMB Project, 2009). The required data consist of an observed catch time series, information on relative abundance and optionally information on age-composition or size composition. Absent composition information it is not possible to estimate selectivity parameters and therefore must be specified *a priori*. The model assumes von Bertalanffy type growth, and that fecundity is proportional to the mature weight at age.

Estimated parameters include the unfished equilibrium recruitment and recruitment compensation rate for the Beverton-Holt stock recruitment model. Average recruitment and average fishing mortality rates are estimated and the corresponding annual rates based on estimated annual deviations. Optionally selectivity parameters are estimated for fisheries and other independent series for which composition information is available.

Table A1. Symbols and descriptions of data variables and parameters used in SCAM.

| Data | | Estimated parameters ($\Theta$) | |
|---|---|---|---|
| $c_t$ | Observed catch | $R_o$ | Unfished age-1 recruits |
| $i_t$ | Relative abundance | $\kappa$ | recruitment compensation |
| $Q_{a,t}$ | Survey age proportions | $\bar{R}$ | Average recruitment |
| $P_{a,t}$ | Fishery age proportions | $\bar{F}$ | Average fishing mortality |
| Life-history info | | $w_{a=1,t}, w_{a,t=1}$ | Recruitment deviations |
| $M$ | instantaneous natural mortality rate | $f_t$ | Fishing mortality deviations |
| $L_\infty, k$ | von Bertalanffy growth parameters | $\psi_a$ | fishery selectivity coefficients |
| $a, b$ | allometry for length-weigth, ($w = al^b$) | $a_h, \gamma_h$ | survey selectivity parameters (logistic) |
| $\hat{a}, \hat{\gamma}$ | age-at-maturity parameters (logistic) | | |

SCAM assumes fishing and natural mortality occur simultaneously, predicted annual catch-at-age and total catch is based on the Baranov catch equation. The initial numbers-at-age are based on average recruitment, age-specific recruitment anomalies and the natural mortality rate. A plus group is also assumed for the oldest age-class. Selectivity is based on either a logistic function, or estimated age-specific selectivity coefficients. In the latter case, two components are added to the objective function: a likelihood for the 2nd differences in the log of the age-specific selectivity coefficients (this ensures a sigmoid like structure of the estimated selectivity coefficients) and a penalty on the amount of dome-shaped selectivity allowed in the estimated coefficients.

For fitting to relative abundance information, the conditional maximum likelihood estimate of the catchability coefficient is used to scale the residuals (see Walters and Ludwig, 1994 for more information). Each of the residual components for the catch, survey or cpue series and recruitment deviations are assumed to be lognormal and are weighted in the objective function by user specified variances. For composition information, SCAM departs from the traditional multinomial or normal likelihoods and uses a multivariate logistic likelihood that is weighted by the conditional maximum likelihood estimate of the variance (see Schnute and Richards 1995 for more information).

Reference points are based on Maximum Sustainable Yield (MSY) assuming the Beverton-Holt model. The fishing mortality rate that maximizes yield is calculated numerically using a Newton-Raphson approach to solve the instantaneous catch equation (see Martell, et. al., 2008 for more information) assuming steady state conditions. Given an estimate of Fmsy, other reference points are derived based on equilibrium recruitment at Fmsy and per recruit incidence functions (e.g., see Figure 1 in Martell, et. al. 2008).

Table A2. Model equations used in SCAM.

Mortality

$F_{a,t} = \bar{F} f_t \psi_a$ Fishing mortality at age $a$ in year $t$

$Z_{a,t} = M + F_{a,t}$ Instantaneous total mortality

$S_{a,t} = \exp(-Z_{a,t})$ Annual survival rate

Constraints

$$\sum_t f_t = 0$$

Initial states

$$N_{a,t} = \bar{R}\exp(w_{a,t}), \quad \forall t, a = 1$$
$$N_{a,t} = \bar{R}\exp(w_{a,t} - M(a-1)), \quad \forall a, t = 1$$
$$N_{a,t} = \frac{N_{a,t}}{1-\exp(-M)}, \quad a = A, t = 1$$

Dynamic states

$$N_{a+1,t+1} = N_{a,t}S_{a,t}, \quad 1 < a < A, 1 < t < T$$
$$N_{a,t+1} = N_{a,t}S_{a,t}, \quad a = A, 1 < t < T$$

Baranov catch equation

$C_{a,t} = \frac{N_{a,t}F_{a,t}(1-S_{a,t})}{Z_{a,t}}$ Catch-at-age numbers

$\hat{P}_{a,t} = \frac{C_{a,t}}{\sum_a C_{a,t}}$ Catch-at-age numbers

$\hat{c}_t = \sum_a C_{a,t} w_a$ Total catch in weight

Survey selectivity

$$v_a = [1 + \exp((\hat{a} - a)/\hat{\gamma}]^{-1}$$

Abundance vulnerable to survey

$$V_{a,t} = N_{a,t} v_a$$

$$\hat{Q}_{a,t} = \frac{V_{a,t}}{\sum_a V_{a,t}}, \quad \text{survey age proporitons}$$

$$\hat{i}_t = \sum_a V_{a,t} w_a, \quad \text{predicted survey biomass}$$

Residuals

$$\epsilon_t = \ln(i_t) - \ln(\hat{i}_t) - \frac{1}{N} \sum_{t \in i_t} \left[ \ln(i_t) - \ln(\hat{i}_t) \right]$$

Unfished spawning biomass ($B_o$)

$$\phi_b = \sum_{a=1}^{a=\infty} w_a m_a e^{-M(a-1)}, \quad \text{spawning biomass per recruit}$$

$$B_o = R_o \phi_b, \quad \text{unfished spawning stock biomass}$$

Beverton-Holt model

$$B_t = \sum_a N_{a,t} w_a m_a, \quad \text{spawning biomass}$$

$$R_{t+1} = \frac{\kappa R_o B_t}{B_o + (\kappa - 1) B_t} e^{\delta_t}, \quad \text{age-1 recruits}$$

Residuals

$$\delta_t = \ln(N_{1,t}) - \ln(R_t)$$

Catch, survey, recruitment

$$\ell_c = 0.5 n \ln(\sigma_c) + \sum_{t=1}^{T} \frac{(c_t - \hat{c}_t)^2}{2\sigma_c^2}$$

$$\ell_i = 0.5 n \ln(\sigma_i) + \sum_{t=1}^{T} \frac{\epsilon_t^2}{2\sigma_i^2}$$

$$\ell_\delta = 0.5 n \ln(\sigma_\delta) + \sum_{t=2}^{T} \frac{\delta_t^2}{2\sigma_i^2}$$

Curvature for selectivity coefficients

$$\ell_\psi = 0.5(A-2) \ln(\sigma_\psi) + \sum_{a=3}^{A} \frac{(\psi_a - 2\psi_{a-1} + \psi_{a-2})^2}{2\sigma_\psi^2}$$

Residuals for age proportions

$$\nu_{a,t} = \ln(Q_{a,t}) - \ln(\hat{Q}_{a,t}) - \frac{1}{A}\sum_{a=1}^{A}\ln(Q_{a,t}) - \ln(\hat{Q}_{a,t})$$

$$\eta_{a,t} = \ln(P_{a,t}) - \ln(\hat{P}_{a,t}) - \frac{1}{A}\sum_{a=1}^{A}\ln(P_{a,t}) - \ln(\hat{P}_{a,t})$$

Likelihoods for age proportions

$$\ell_Q = (A-1)T\ln\left(\frac{1}{(A-1)T}\sum_{a=1}^{A}\sum_{t=1}^{T}\nu_{a,t}^2\right)$$

$$\ell_P = (A-1)T\ln\left(\frac{1}{(A-1)T}\sum_{a=1}^{A}\sum_{t=1}^{T}\eta_{a,t}^2\right)$$

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