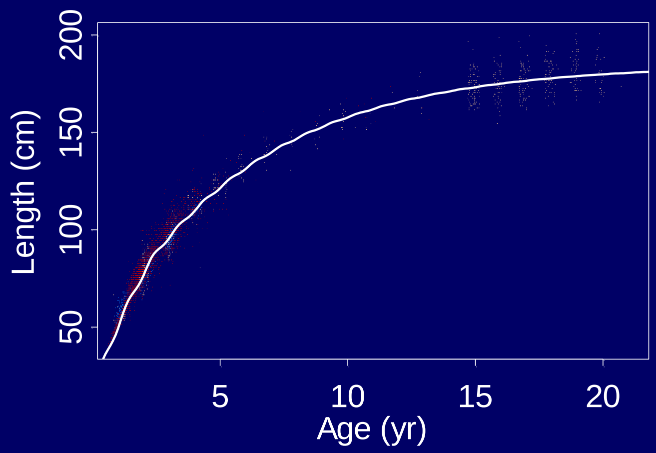
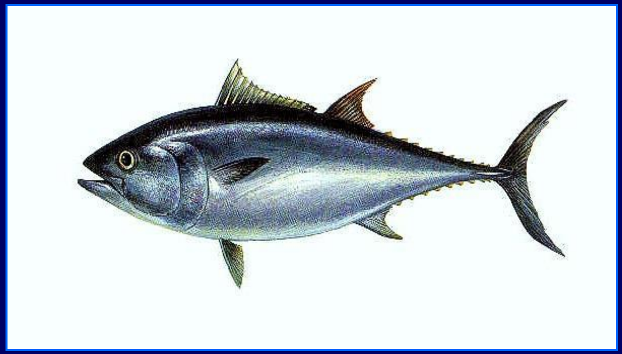
A comprehensive growth model for tuna in the Indian Ocean

Paige Eveson
CSIRO Marine and Atmospheric Research





IOTC Tagging Symposium, Grand Baie, Mauritius, 30 Oct – 2 Nov 2012

Acknowledgments

- Model development: Tom Polacheck, Geoff Laslett
- Otolith data: Fany Sardenne, Gael LeCroizier, and other readers
- Tagging data: Julien Million

Why do we need growth models?

- fundamental to stock assessments for estimating age composition of catch
- changes in growth have important implications about stock status (i.e. density dependence)

Sources of growth information

Three common sources:

- Change in length and time at liberty data from tag-recapture experiments
- Direct age and length data obtained from hard-part analyses (e.g. otoliths)
- Length-frequency data from commercial catches - *not used here*

A general growth curve

For a fish of length l and age a:

$$l(a) = L_\infty f(a - a_0; \theta)$$

L_∞ is the asympotic length

f is a monotonic increasing function with parameter set $\{\theta, a_0\}$

$f = 0$ when $a = a_0$ (i.e. a_0 is the theoretical age of length 0)

$\lim_{a \to \infty} f = 1$

Specific growth functions

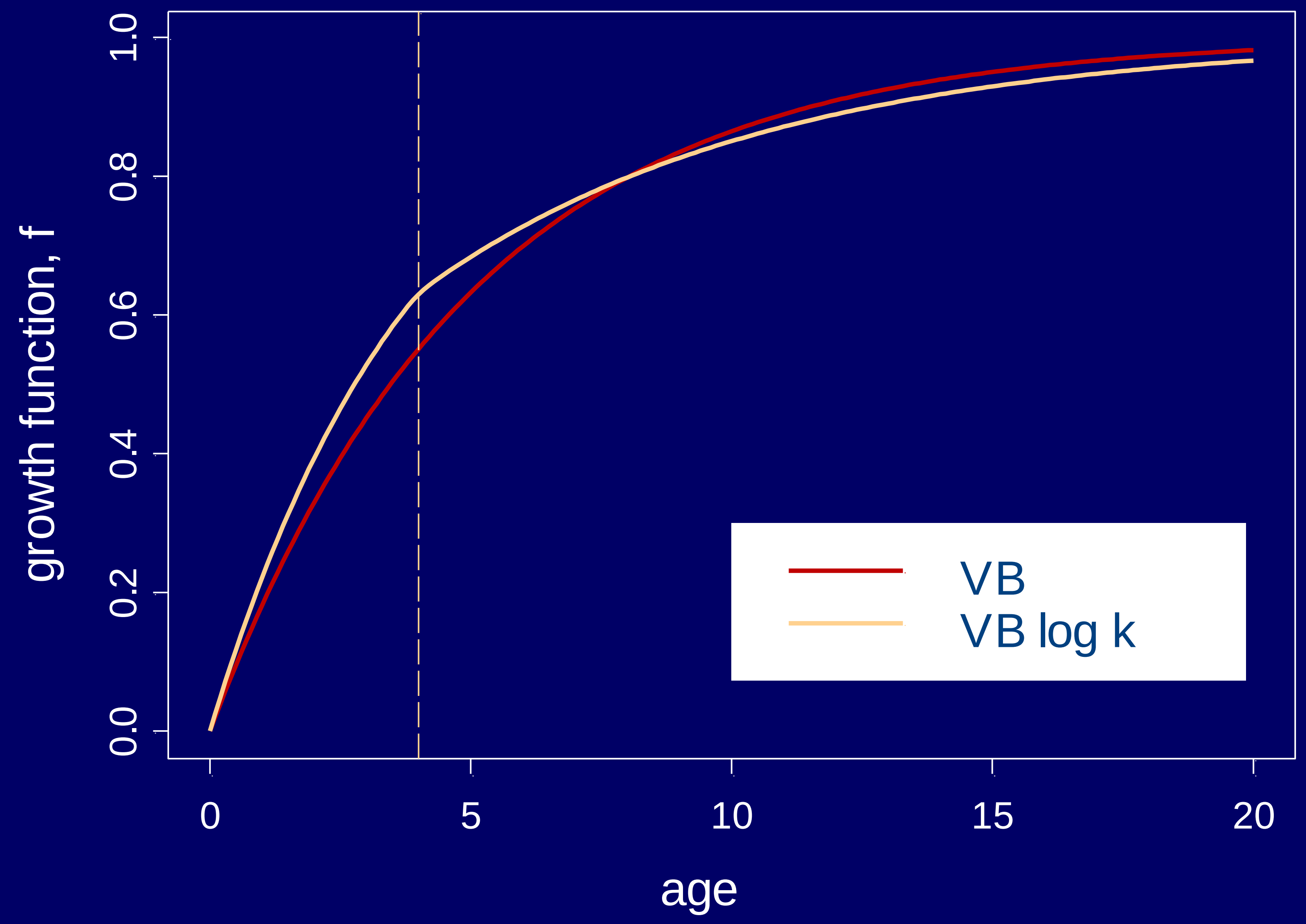
1) VB (von Bertalanffy) function:

$$f(a-a_0;\theta)=1-e^{-k(a-a_0)} \quad \text{where } \theta=\{k\}$$

2) VB log k function:

$$f(a-a_0;\theta)=1-e^{-k_2(a-a_0)}\left\{\frac{1+e^{-\beta(a-a_0-\alpha)}}{1+e^{\alpha\beta}}\right\}^{-(k_2-k_1)/\beta}$$

where $\theta=\{k_1,k_2,\alpha,\beta\}$



growth function, f
0.0
0.2
0.4
0.6
0.8
1.0
0
5
10
15
20
age
VB
VB log k

Model description

- Maximum likelihood method with separate likelihood for each data source
- Data sets independent so multiply together to obtain overall likelihood to be optimized

Tag-recapture likelihood

- Model the joint density of release and recapture lengths
- Age at release unknown so we model it as a random variable, A:
  - here we assume $A \sim \log N(\mu_A, \sigma_A^2)$
- Allow for individual variability in growth by modelling asymptotic length as a random effect, $L_\infty \sim N(\mu_\infty, \sigma_\infty^2)$

Tag-recapture likelihood

For fish i tagged at time t_{1i} with length l_{1i} and unknown age a_{1i}, and recaptured at time t_{2i} with length l_{2i} :

$$l_{1i} = L_{\infty,i} f(A_i;\theta) + \varepsilon_{1i}$$

$$l_{2i} = L_{\infty,i} f(A_i + t_{2i} - t_{1i};\theta) + \varepsilon_{2i}$$

where

$$A_i = a_{1i} - a_0 \sim \log N(\mu_A, \sigma_A^2)$$

$$L_{\infty,i} \sim N(\mu_\infty, \sigma_\infty^2)$$

$$\varepsilon_{1i} \sim N(0, \sigma_s^2)$$

Tag-recapture likelihood

Condition on A, then the joint distribution of l_1 and l_2 is bivariate normal (see Laslett, Eveson & Polacheck 2002 CJFAS 59: 976-986 for exact formulation). Then the unconditional density is:

$$\int h(l_{1i}, l_{2i} \mid a)\, p(a)\, da$$

where

$h(\cdot)$ is the bivariate normal density

$p(\cdot)$ is the density of A (log normal)

Otolith likelihood

For fish i with length l_i and age a_i:

$$l_i = L_{\infty,i} f(a_i - a_0;\theta) + \gamma_i$$

where

$$L_{\infty,i} \sim N(\mu_\infty, \sigma_\infty^2)$$

$$\gamma_i \sim N(0, \sigma_\gamma^2)$$

a_i = age calculated from # of bands in otolith

Otolith (log) likelihood

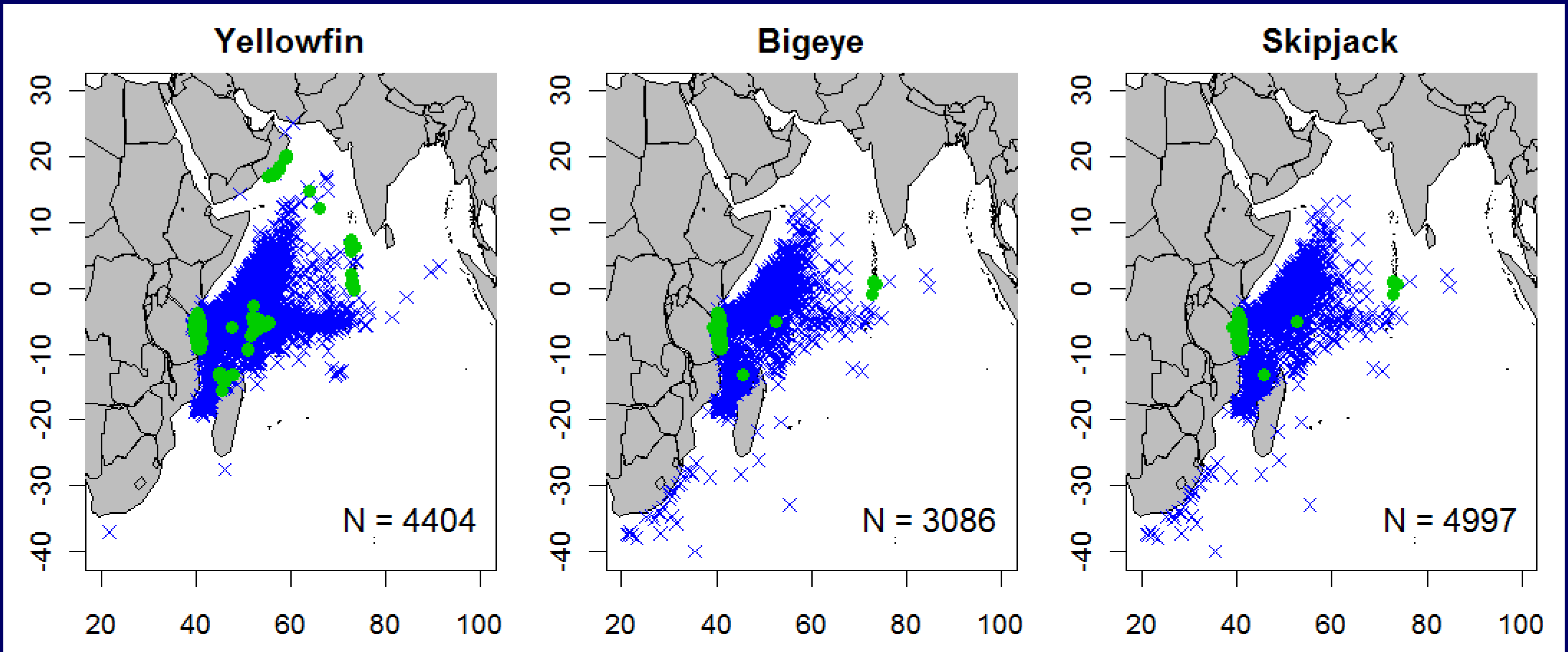
$$\ln(\lambda_{OTO}) = \frac{1}{2}\sum_i \left[\ln\left(2\pi V(l_i)\right) + \frac{\left(l_i - E(l_i)\right)^2}{V(l_i)} \right]$$

where

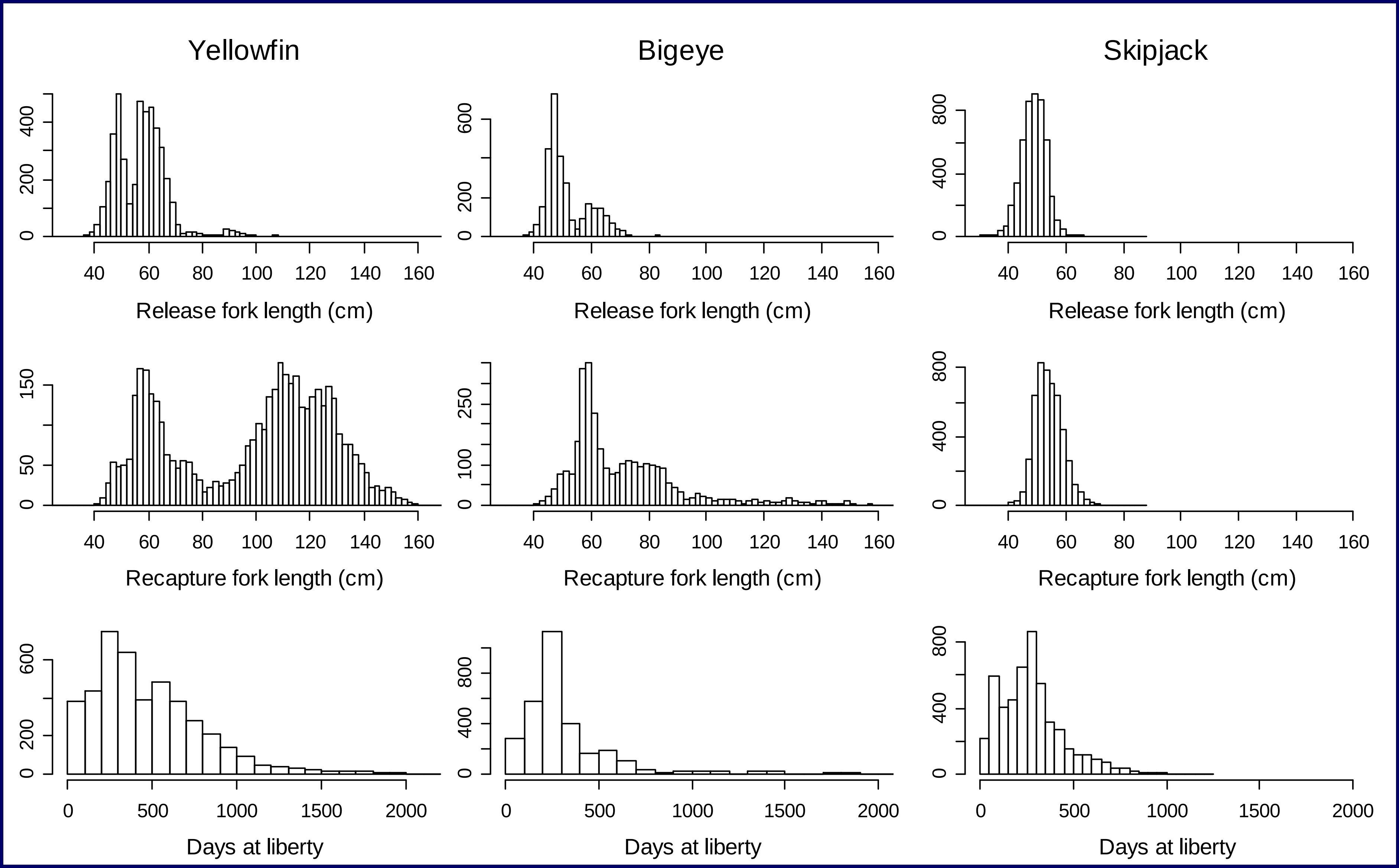
$$E(l_i) = \mu_\infty f(a_i - a_0; \theta)$$

$$V(l_i) = \sigma_\infty^2 f(a_i - a_0; \theta)^2 + \sigma_\gamma^2$$

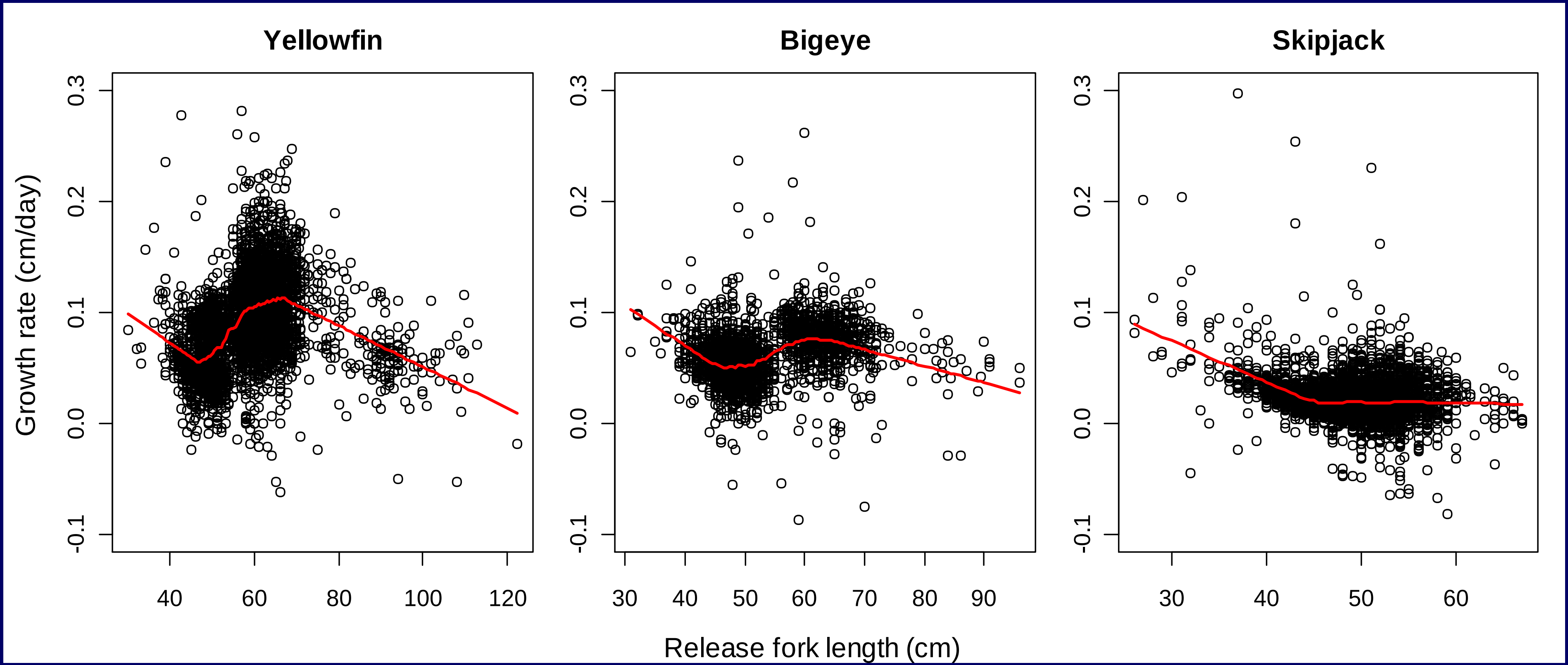
IOTTP tag-recapture data



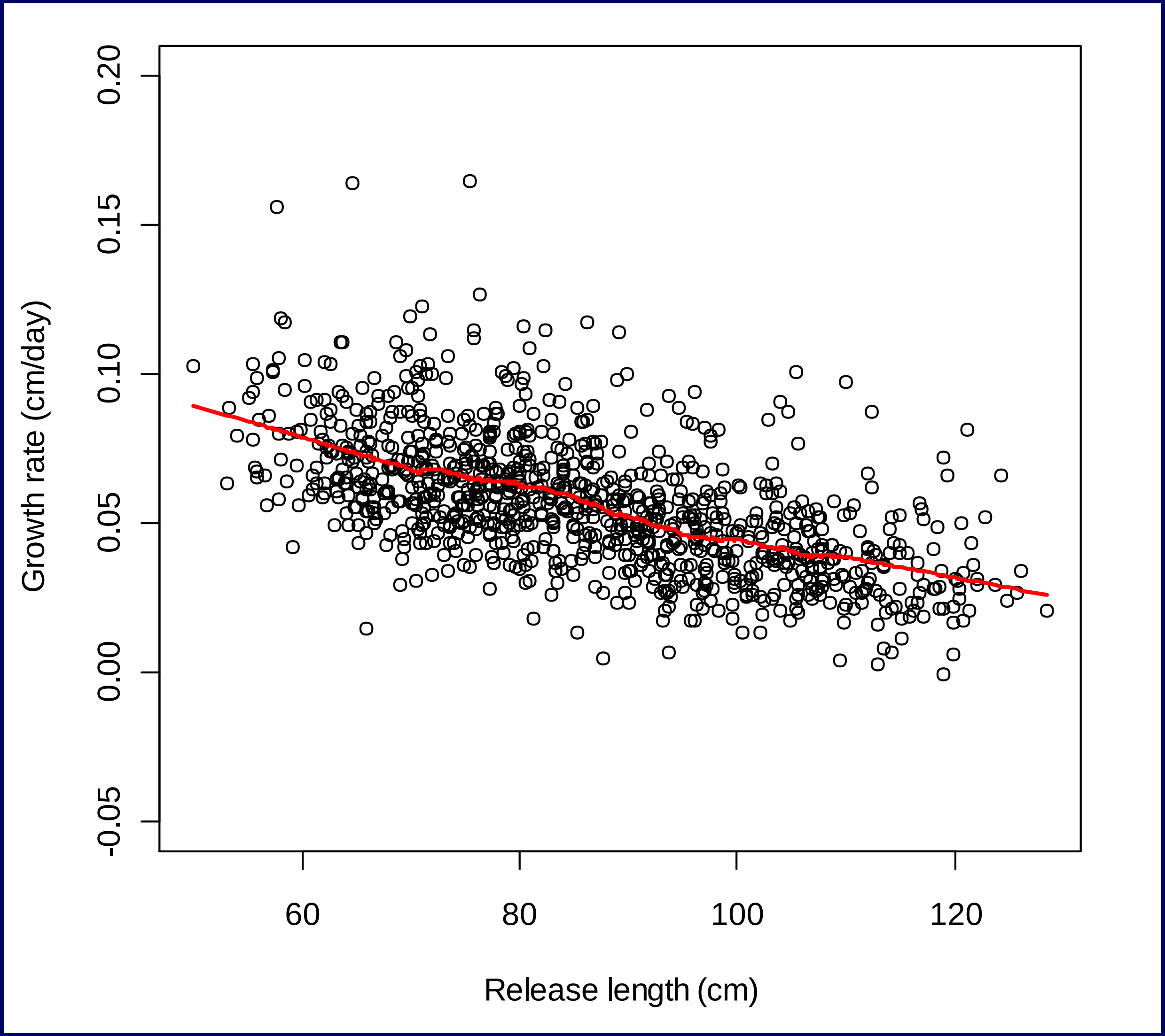
IOTTP tag-recapture data



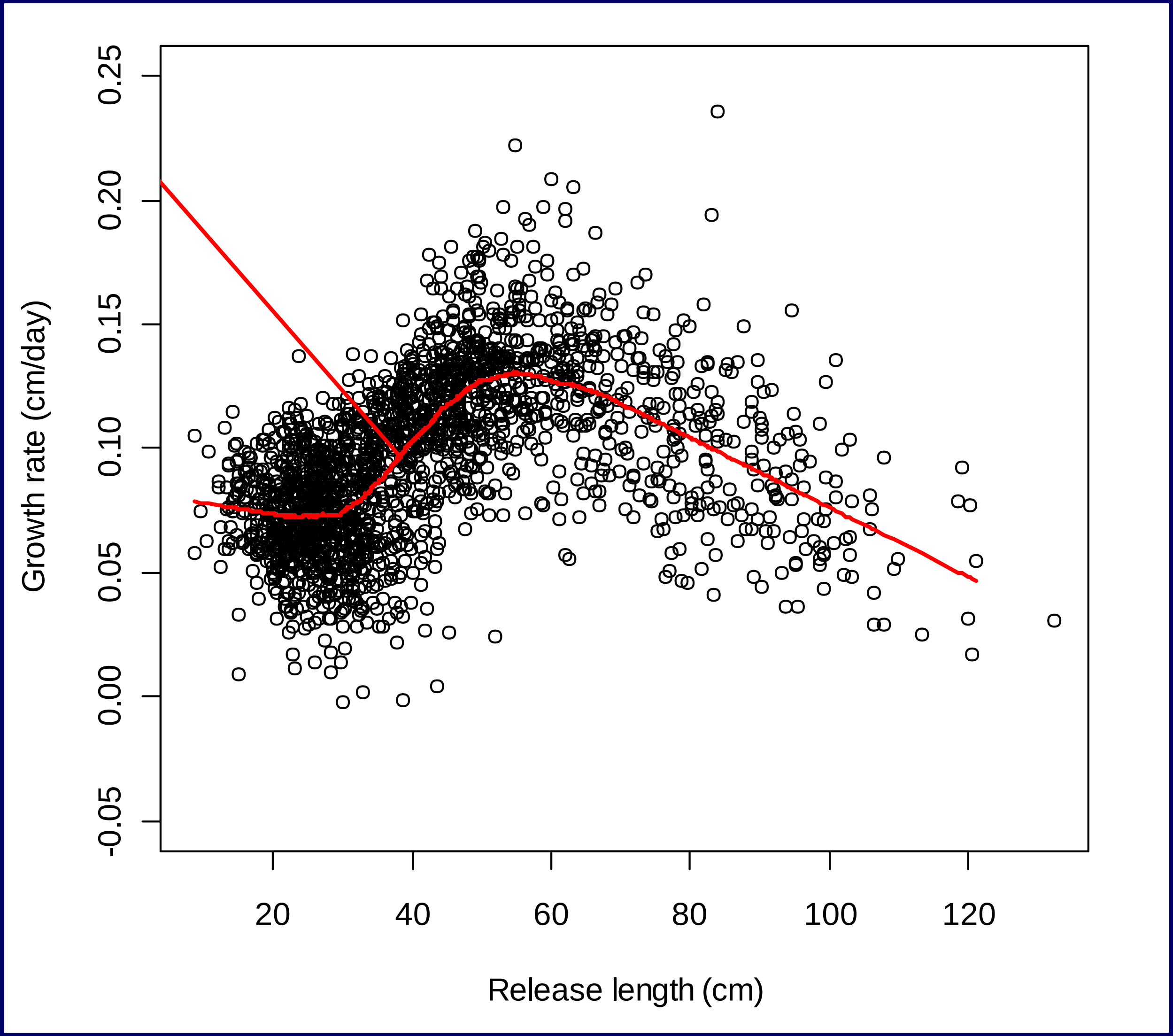
IOTTP tag-recapture data



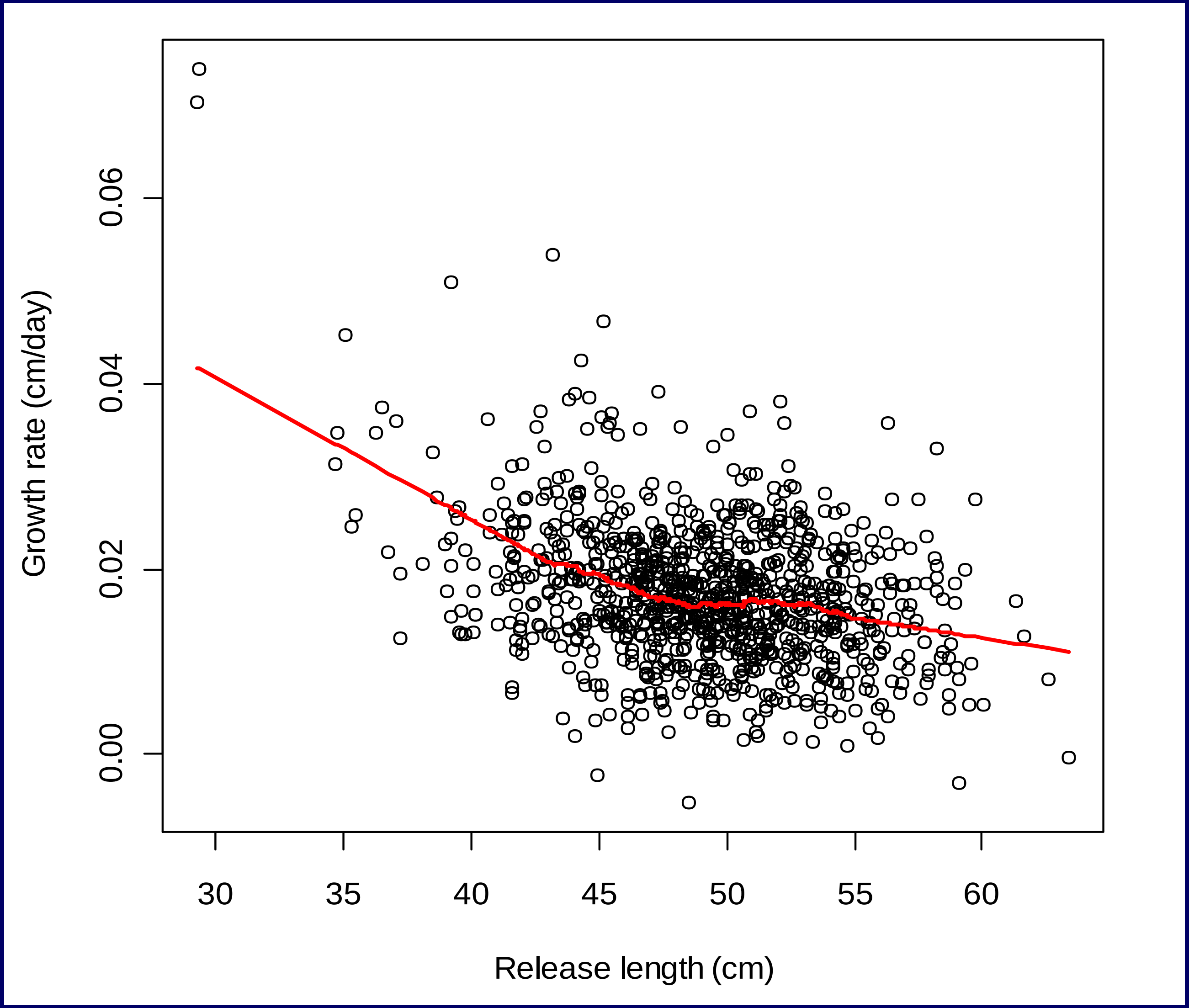
Simulated VB growth



Simulated VB log k growth with bi-modal release lengths & k1<k2

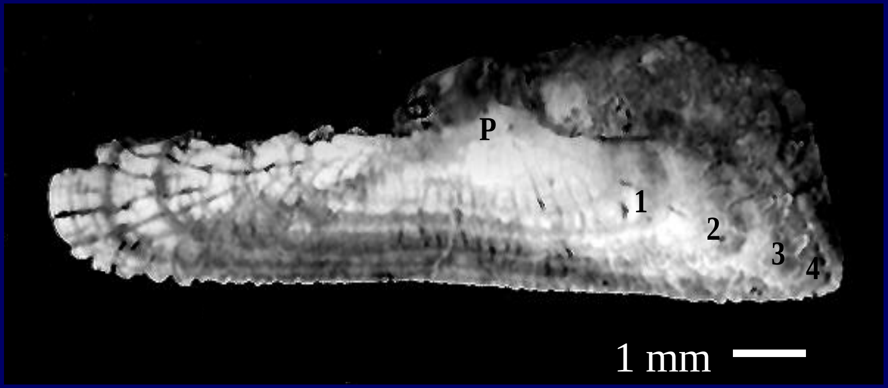


Simulated VB log k growth with uni-modal release lengths & k1>k2

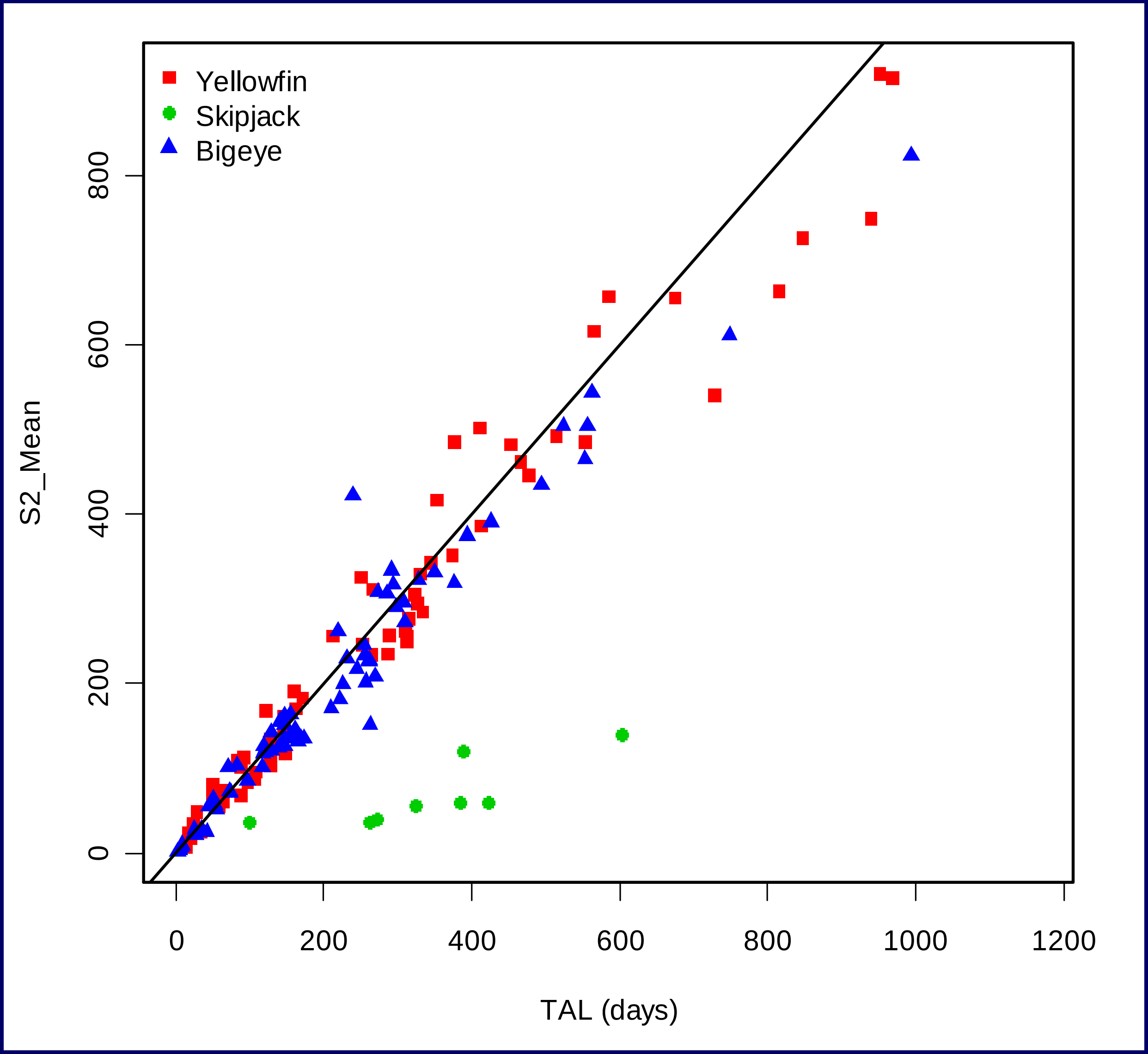


Otolith data

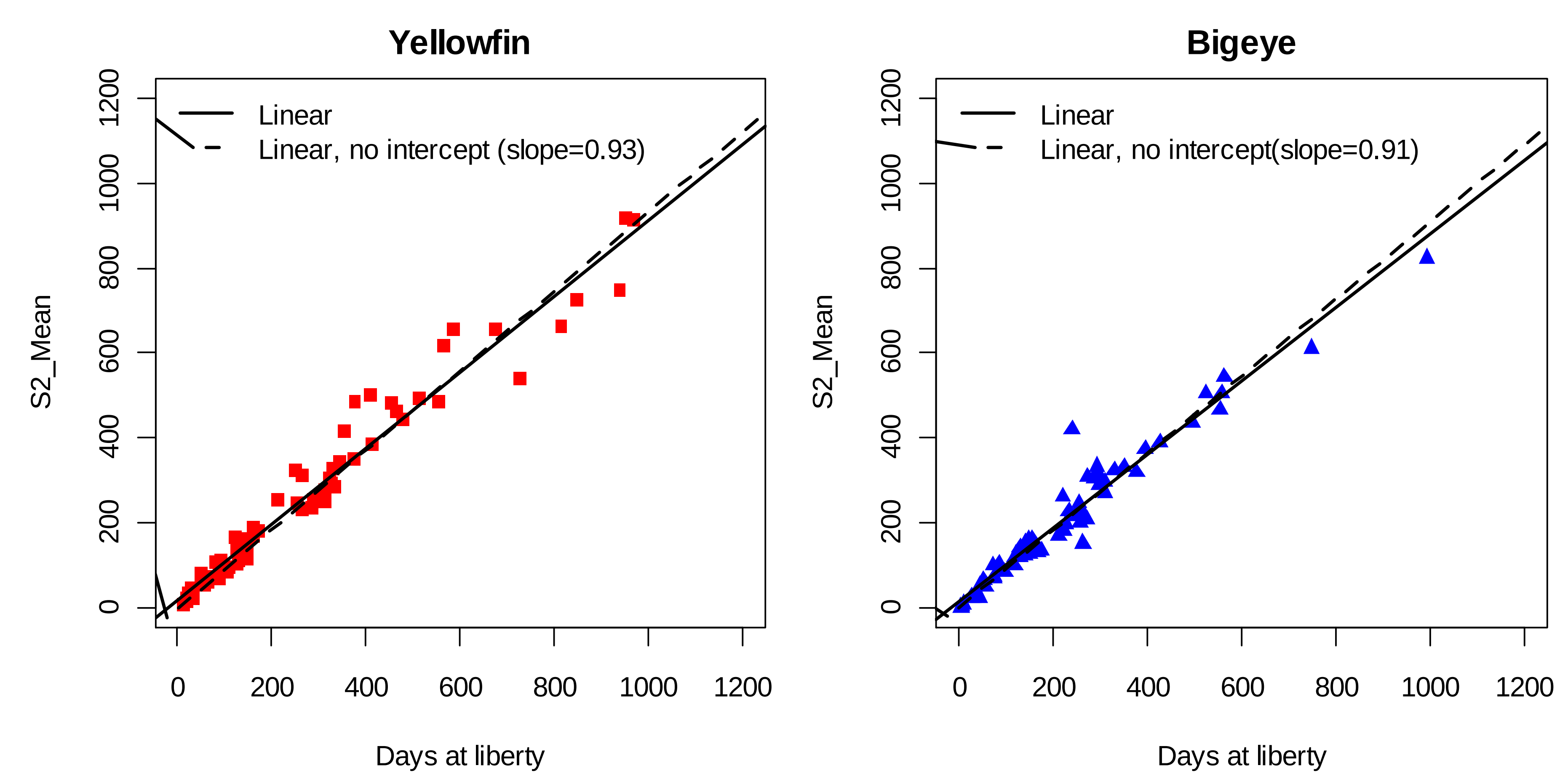
- Verify hypothesis that growth bands are formed daily in otoliths using OTC-marked recaptures:
  - compare counts after OTC mark with number days at liberty
- Correct counts based on relationship between number of counts and days at liberty



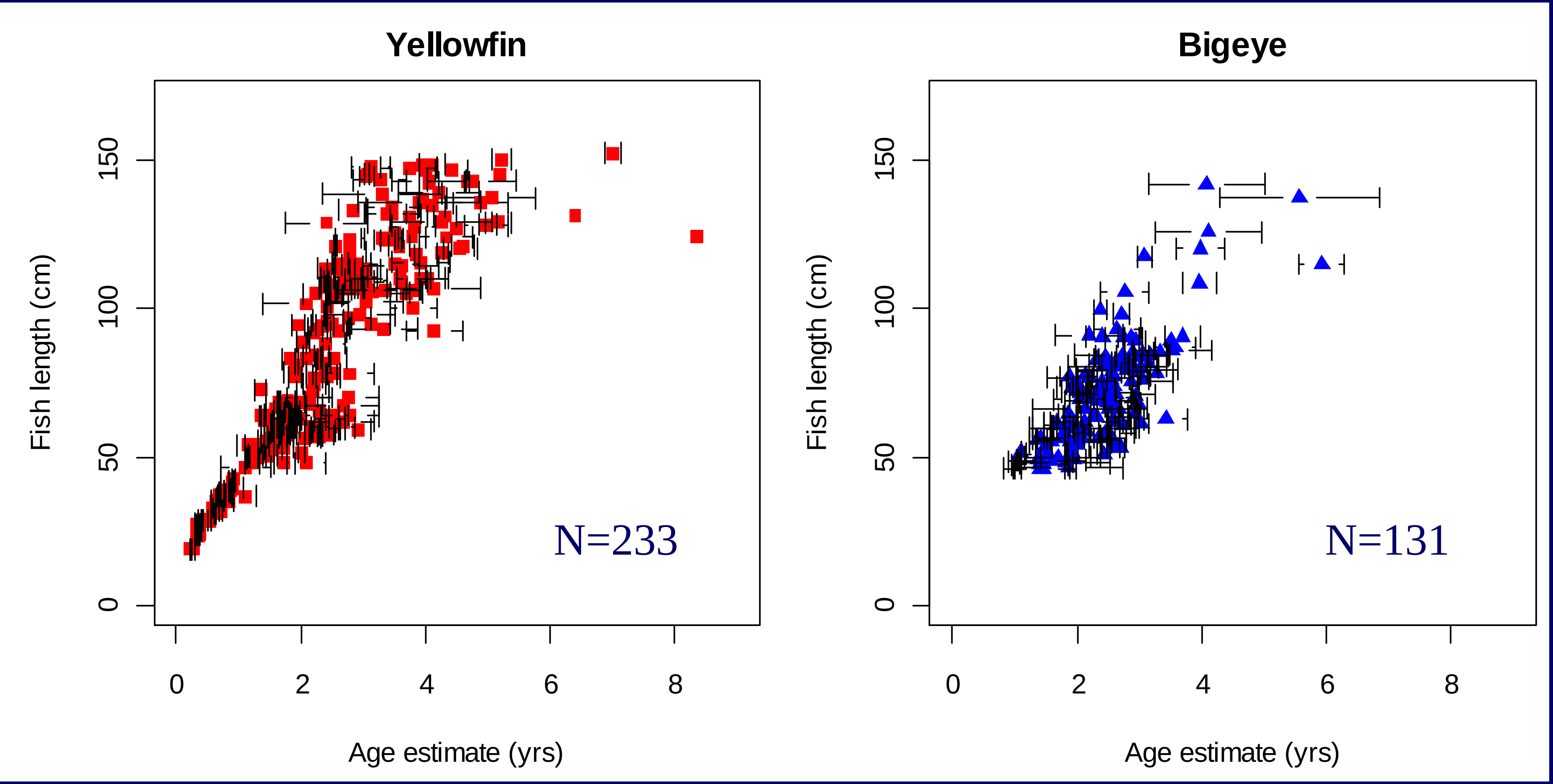
Otolith counts from OTC-marked fish vs. days at liberty



Otolith counts from OTC-marked fish vs. days at liberty



Corrected otolith age and length data



Yellowfin results

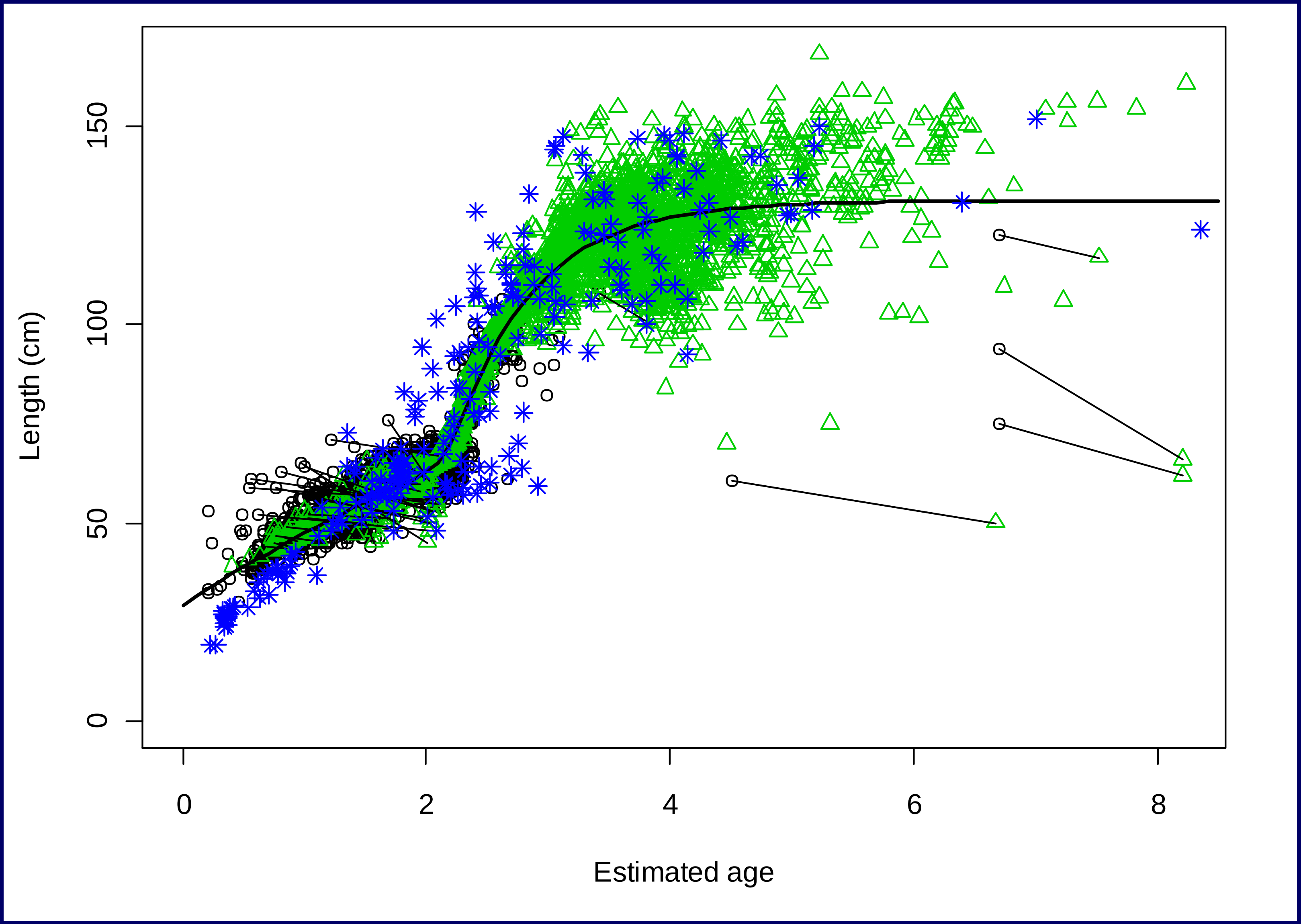
Model	μ∞	σ∞	k1	k2	α	β	μ.A	σ.A	σ.tag	a0	σ.oto
1	132.3	10.1	0.20	1.51	3.5	15.6	1.06	0.14	3.9	-1.3	11.9
2	**145**#	10.4	0.15	0.94	3.8	20^	1.18	0.13	4.3	-1.7	12.8

*Estimate on lower bound

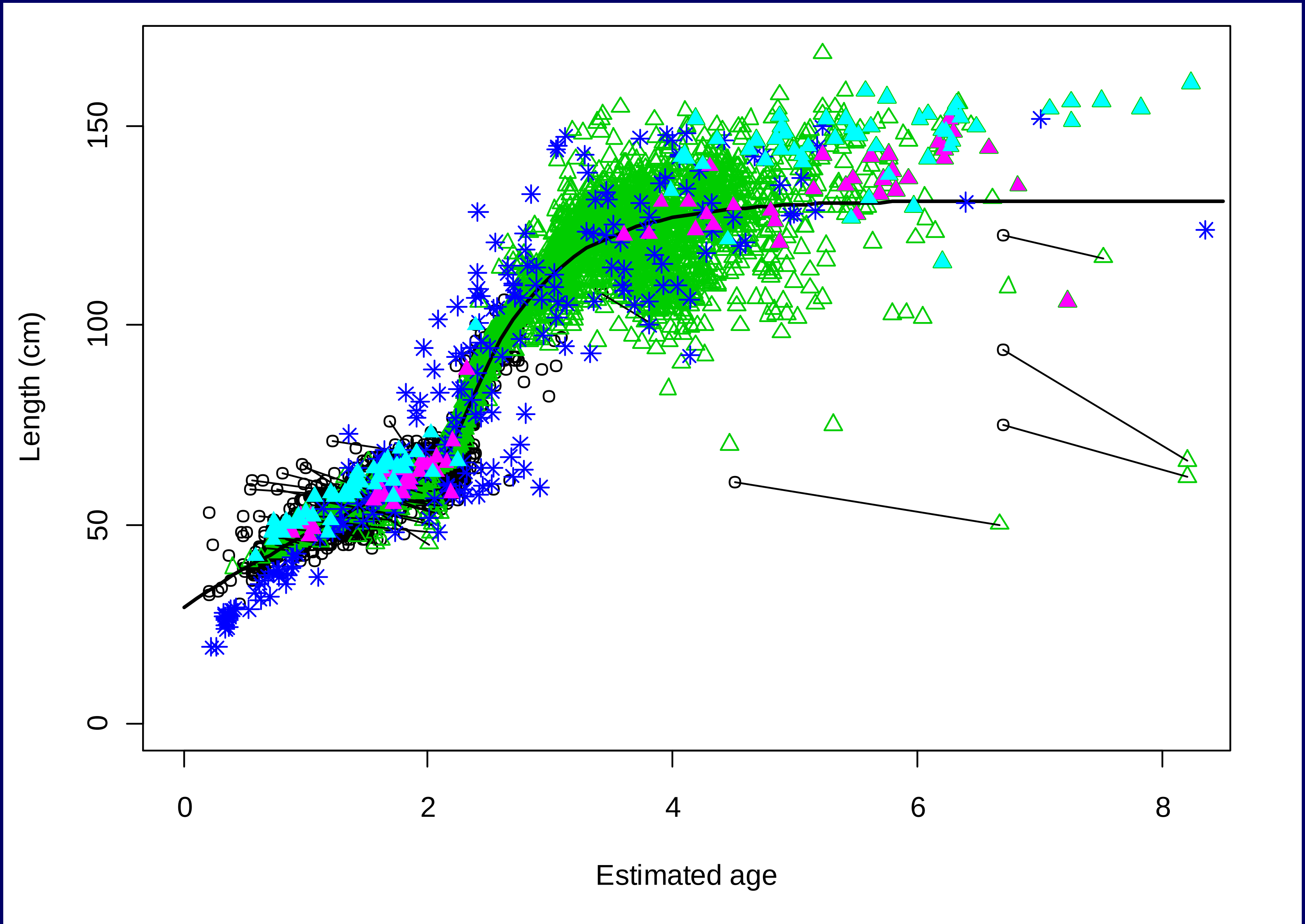
^Estimate on upper bound

Fixed

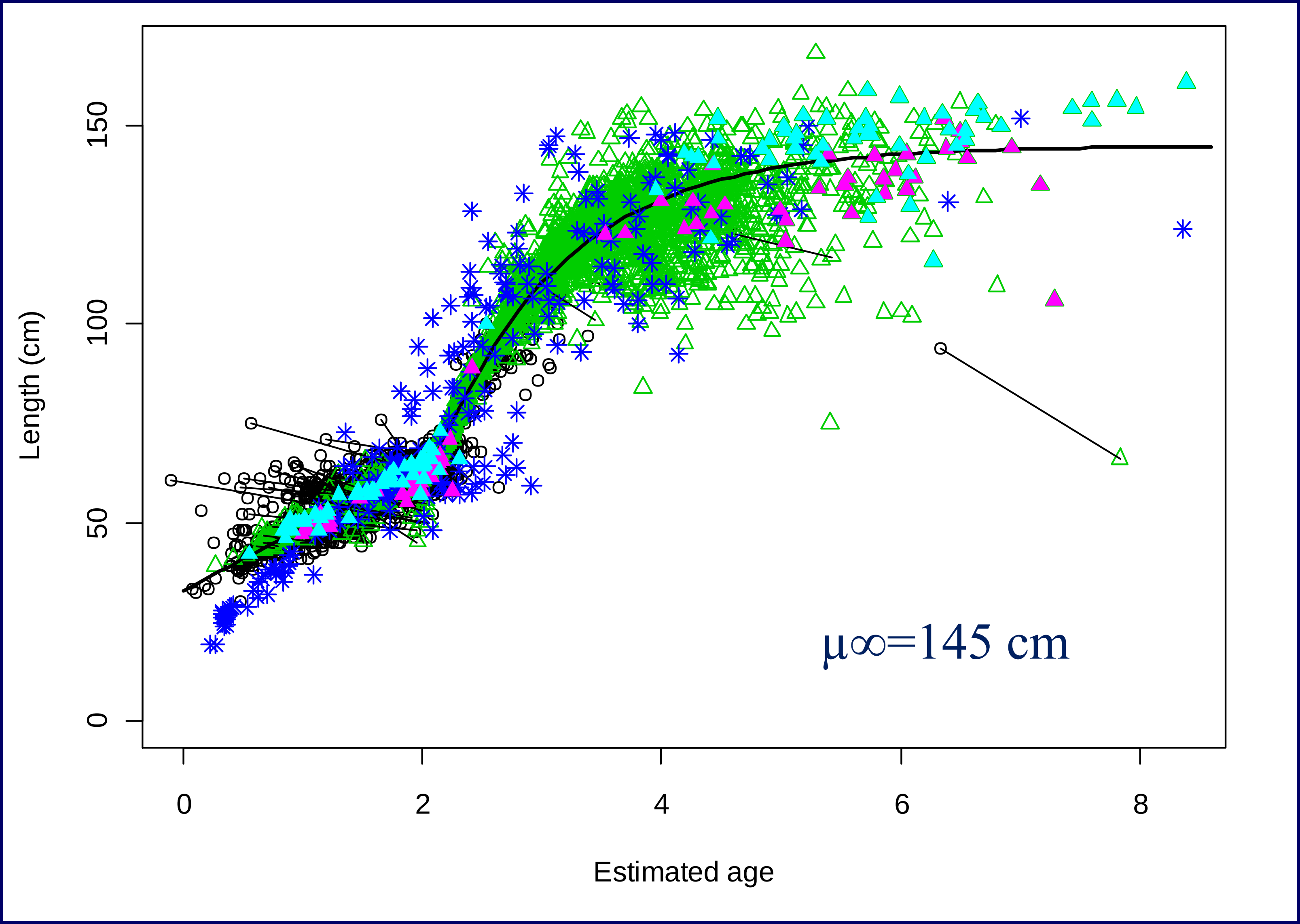
YFT Model 1: fitted growth curve



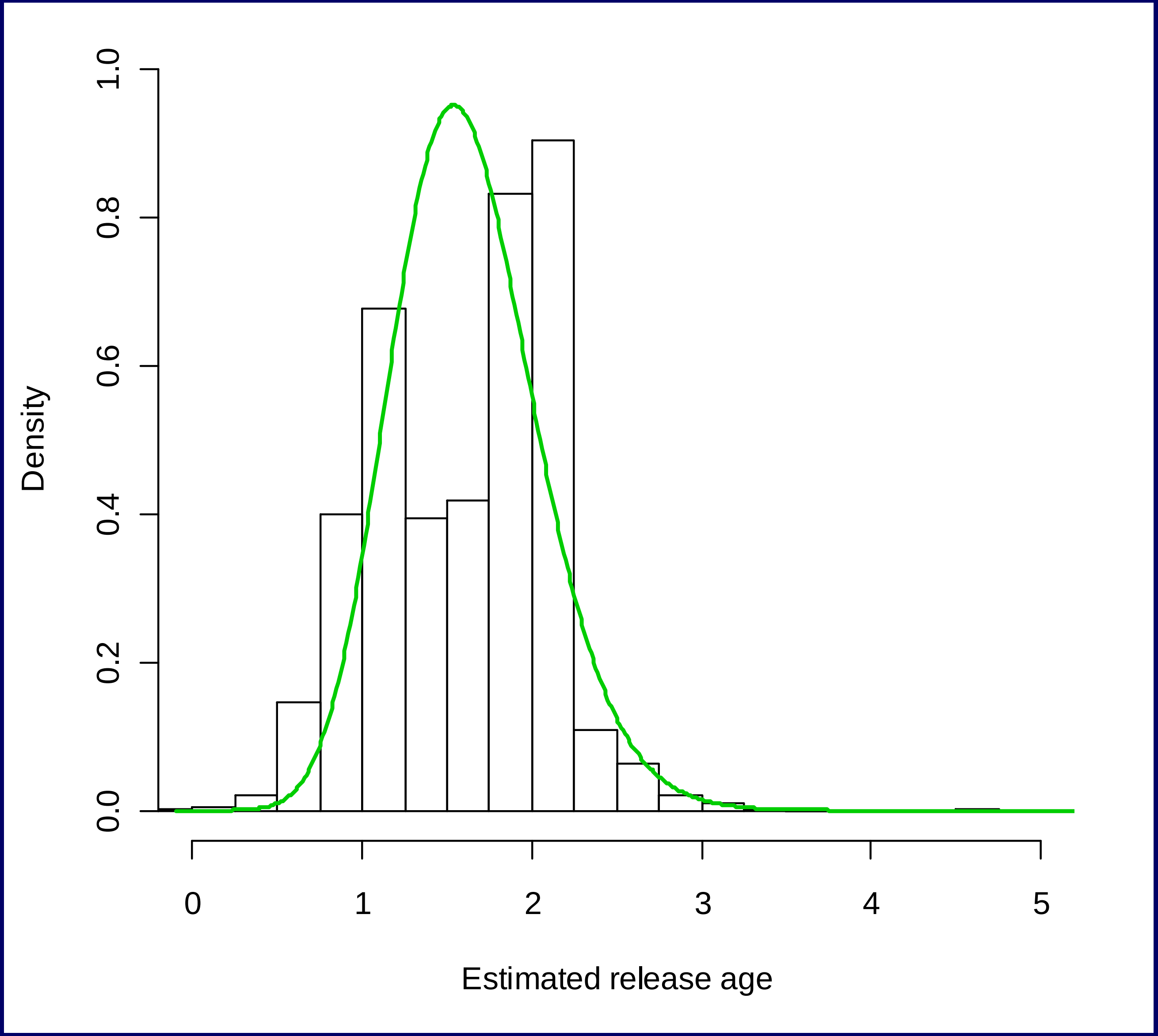
YFT Model 1: fitted growth curve



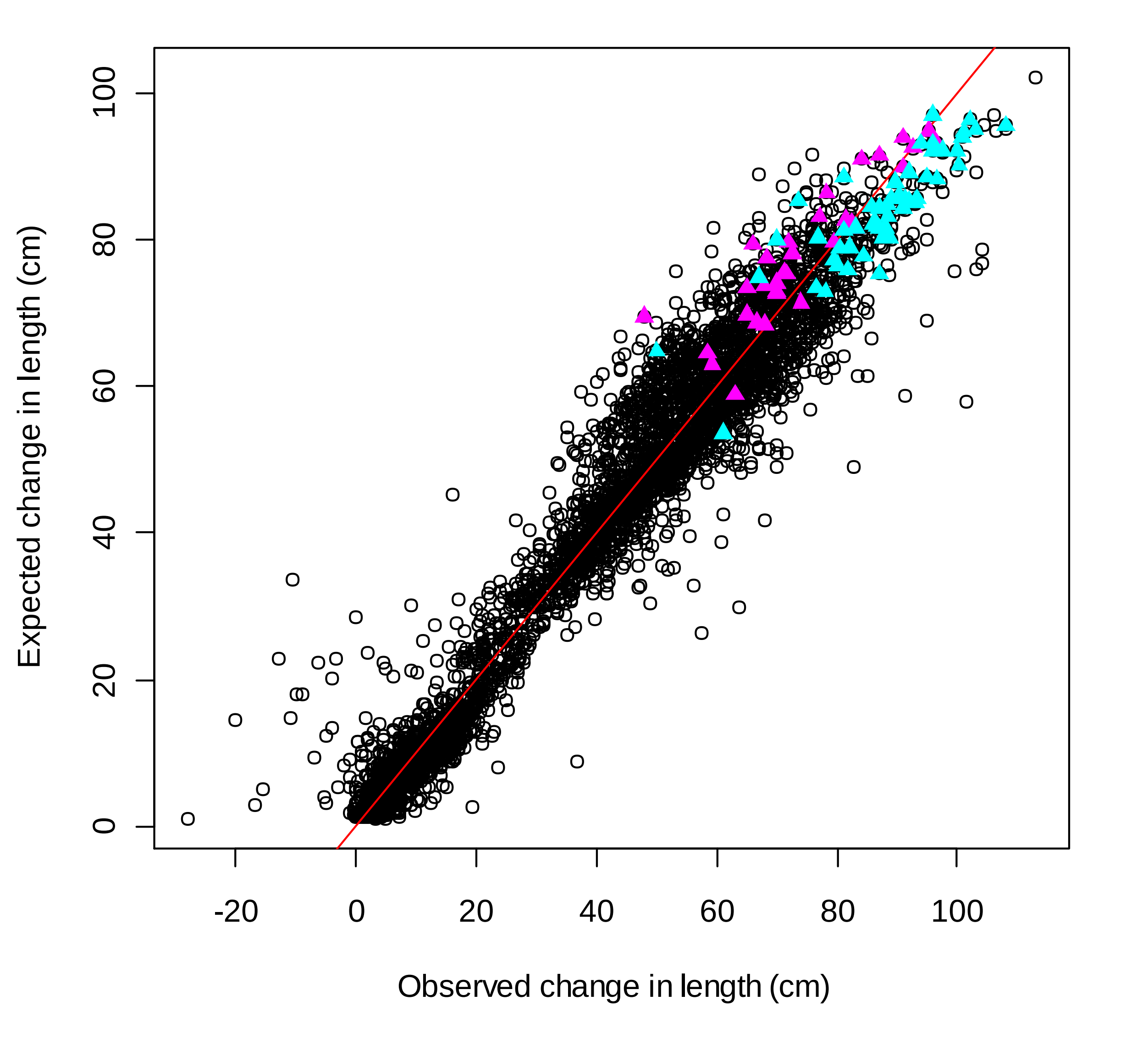
YFT Model 2: fitted growth curve



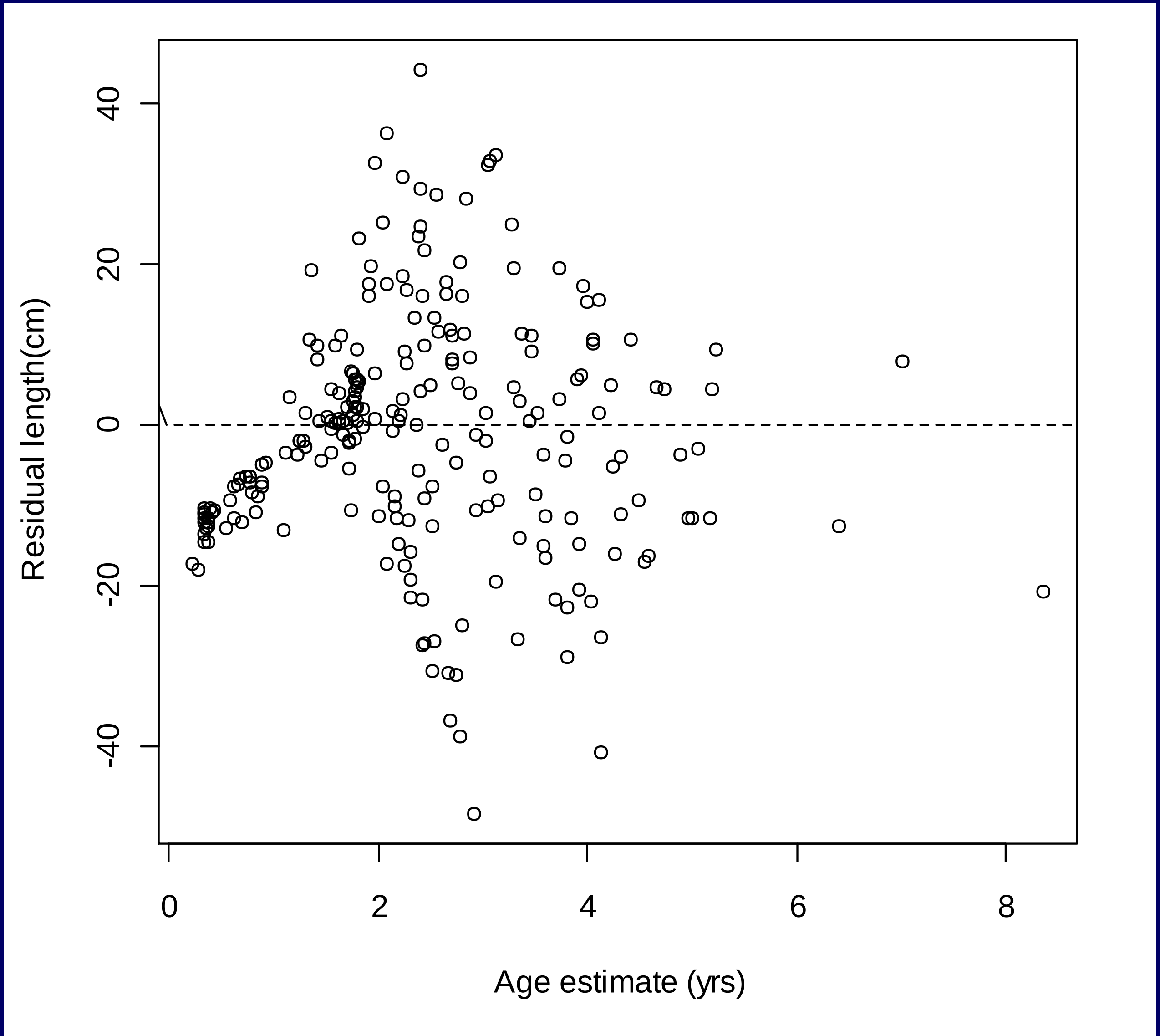
YFT Model 2: tag-recapture fit



YFT Model 2: tag-recapture fit



YFT Model 3: otolith fit



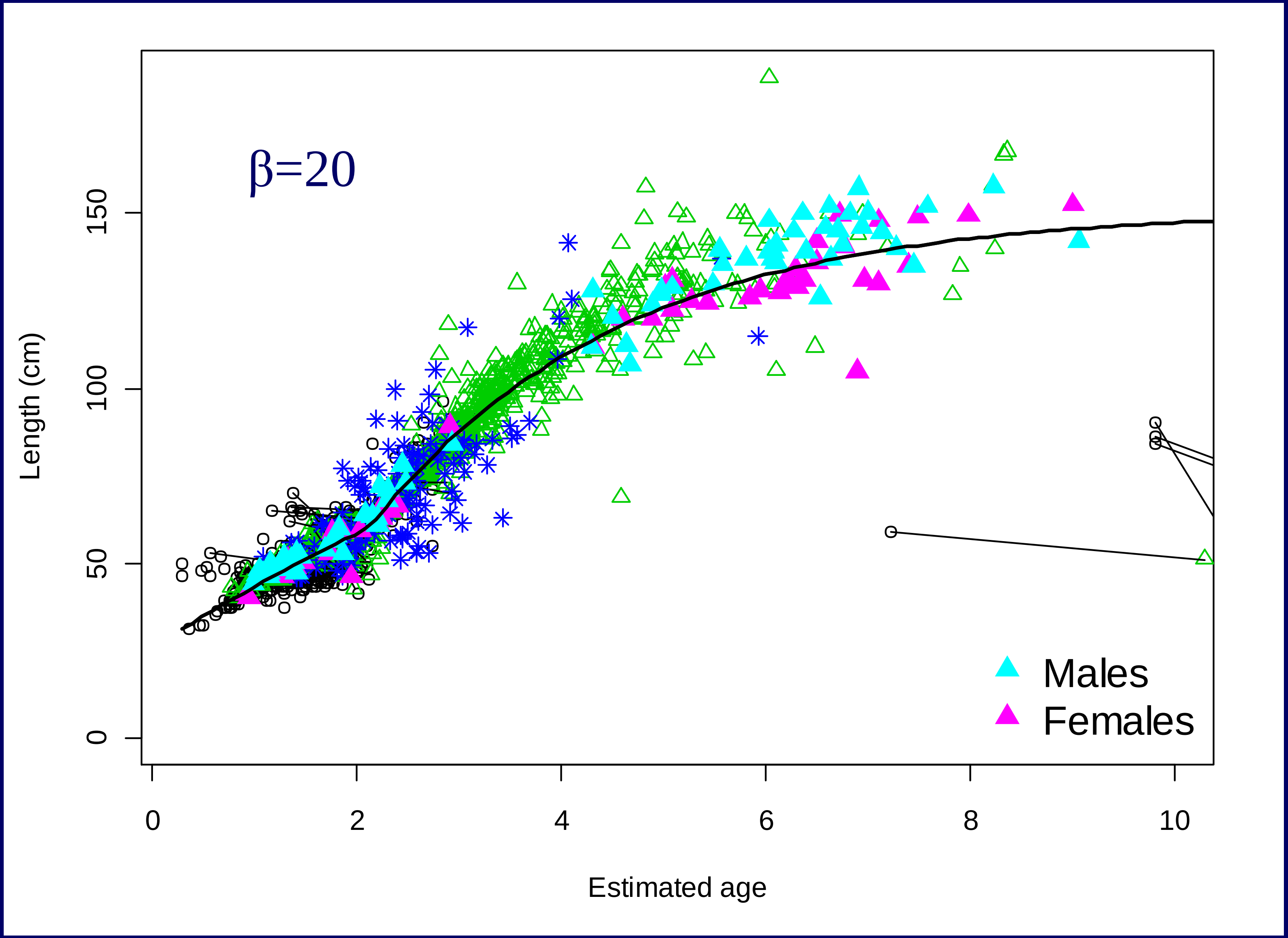
Bigeye results

Model	μ∞	σ∞	k1	k2	α	β	μ.A	σ.A	σ.tag	a0	σ.oto
1	145.5	9.5	0.02	0.55	17.1	2.3	2.8	0.03	2.9	-15*	10.5
2	150.9	11.7	0.15	0.41	3.4	**20#**	1.02	0.14	3.1	-1.2	7.8

*Estimate on lower bound

Fixed

Bigeye Model 2: fitted growth curve

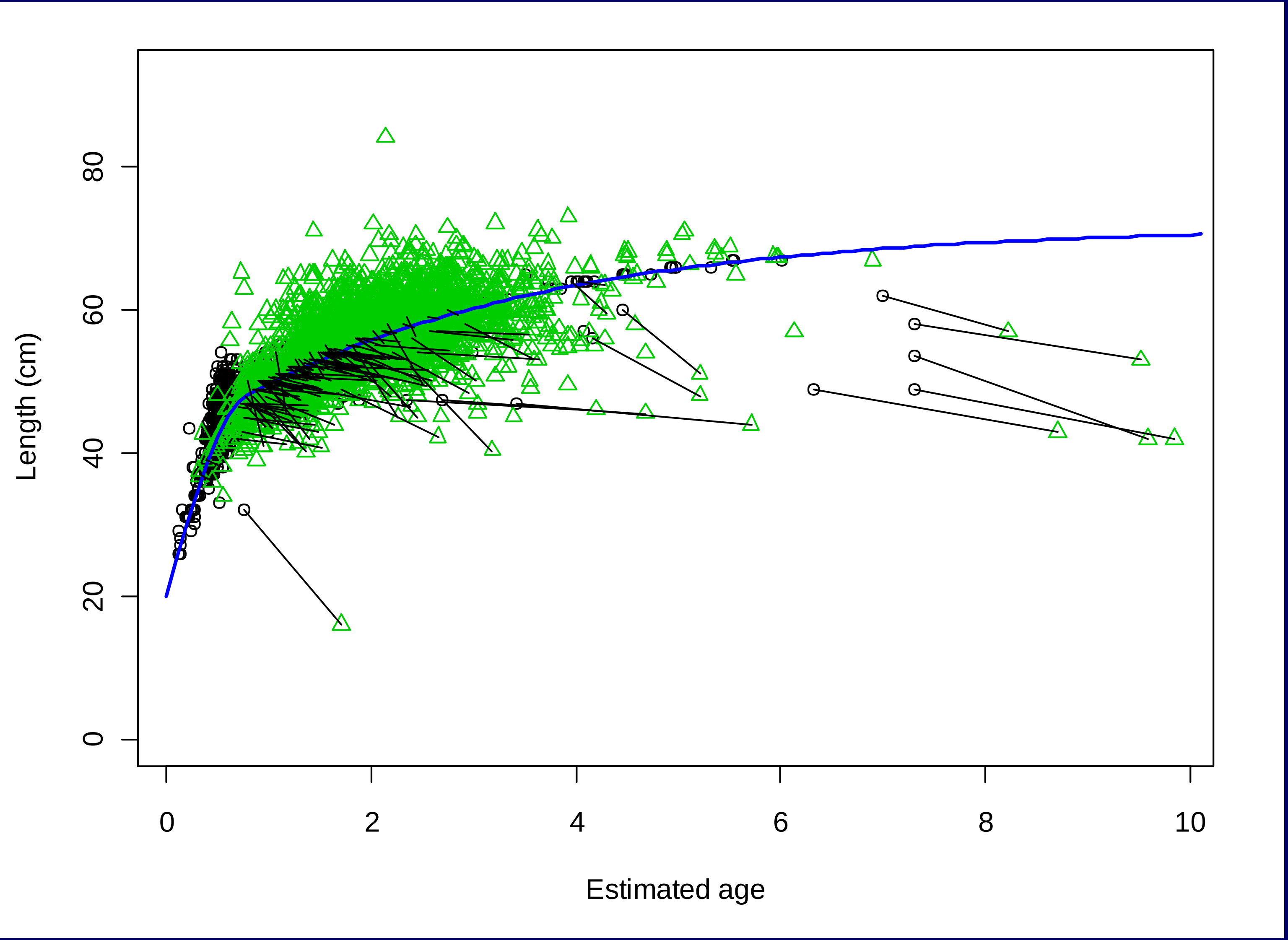


Skipjack results

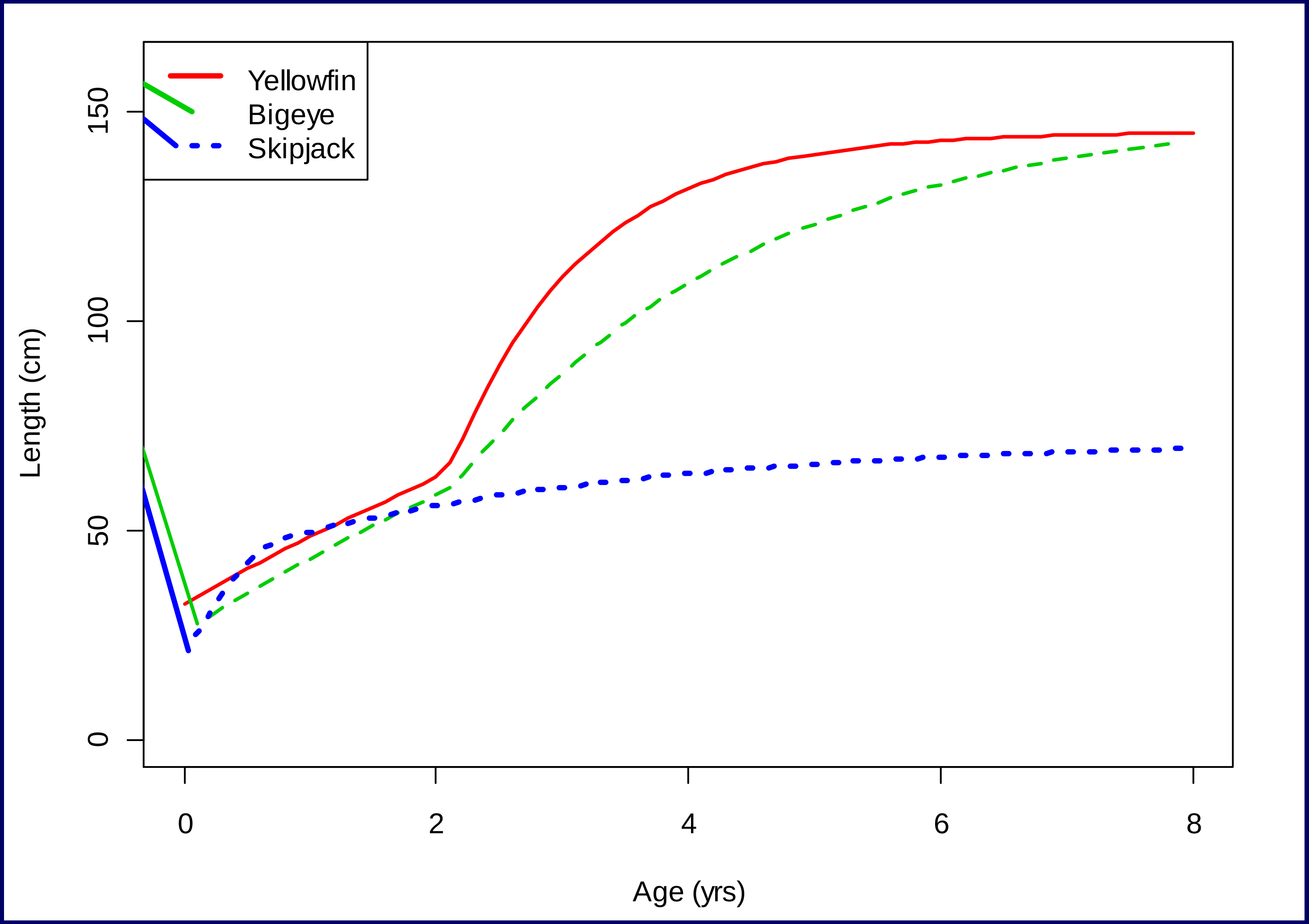
Model	μ∞	σ∞	k1	k2	α	β	μ.A	σ.A	σ.tag	**a0***
1	71.6	4.9	1.12	0.33	0.95	24.8	0.22	0.23	1.7	**-0.3**

* Calculated after fitting the model so that L(0) = 20cm (since we do not get an estimate of a0 from the model with tag-recapture data alone)

Skipjack Model 1: fitted growth curve



Comparison of mean growth curves for the three species



Future Work

- Incorporate error in the ages of the direct aging data
- Obtain realistic standard errors for parameter estimates
  - difficulty getting from inverse Hessian
  - likelihood profiles more reliable but very slow
- Allow for growth parameters to change over time (some work done on this)