

Bayesian state-space production models for the Indian Ocean bigeye tuna (*Thunnus Obesus*) and their predictive evaluation

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Abstract

In stock assessment, it is not straightforward to choose a plausible range of models objectively from several models if different data set are used because it is not possible to use model selection criteria like AIC in these situation. However, as shown in Kell et al. (2016), where a hindcasting approach was proposed, predictive evaluation via cross-validation would be a possible procedure under those circumstances. Here, as an attempt using data for bigeye tuna, we applied a model selection method with predictive evaluation of biomass index to Bayesian state-space production models although the data used is common to the model in this case. Using a selected model, we also assessed the population status of the stock. Non-informative priors were used and posterior samples were generated using a Markov chains Monte Carlo (MCMC) method. The results suggested that F-ratio (2015) is higher than the MSY level (1.17) and B-ratio is lower than 1 (0.76). Given that this analysis has a preliminary nature as stock assessment, the paper may not be so useful for management advice, but this approach could give an opportunity to help in choosing models in future assessment.

1. Introduction

In this paper, we introduce our attempt to selecting the best model (of a range of better models) from the point of view of predictive ability. Here we will apply the approach to Bayesian state-space production models. The reason we chose production model for the first attempt is its robustness and simplicity. Production model only needs catch data and abundance indices like CPUE series, so it is relatively easy to handle.

2. Data

To implement Production Model, we used the annual total catch data (Fig 1) and yearly STD CPUE series that were estimated through analyses with generalized linear models (see Fig 2). Since CPUE in the tropical region had a better relation to catch (Nishida et al, 2011), we chose the joint CPUE (area R1+R2) (Hoyle et al, 2016) for our main exercise but also used Japanese STD CPUE (Tropical) (Matsumoto et al, 2016) for comparison. Though catch and joint CPUE are available before 1960, we use data in the period of 1960 to 2015 to adjust to Japanese CPUE series.

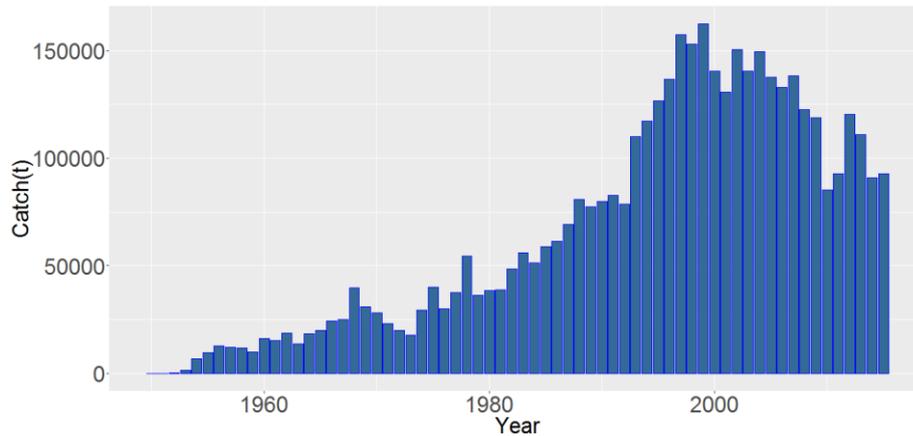


Figure 1. Time series of annual total catch (tons).

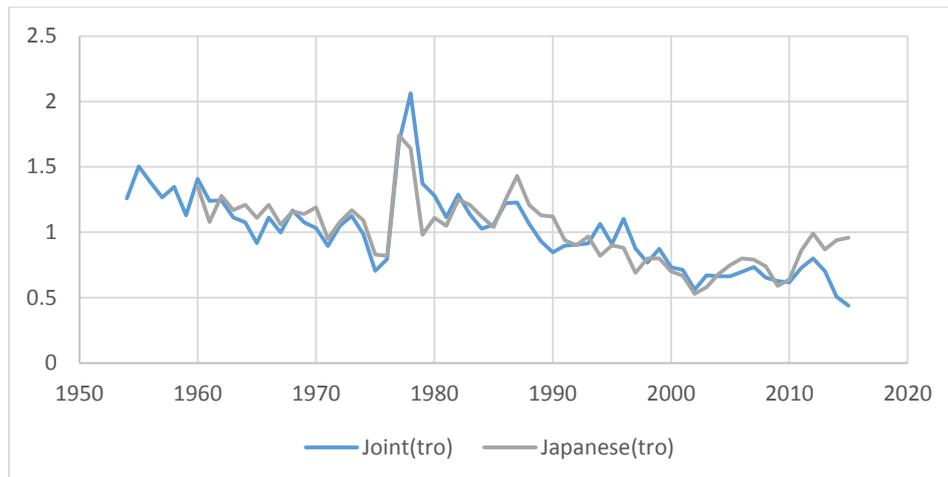


Figure 2. Comparison of Japanese STD CPUE (Tropical) and Joint CPUE (Tropical) series.

3. Methods

3.1 Software

All the analyses in this research were performed with R. For fitting to CPUE data series to Bayesian state-space production models, we used WinBUGS for Gibbs Sampling.

3.2 Models and Statistical procedures

In production model, population has to be considered as whole biomass of single species. Thus we assumed that bigeye tuna in Indian Ocean is a single stock and can't get any estimation related to spawning stock biomass. The general production model equation is:

$$B_{t+1} = B_t + g(B_t) - C_t \tag{1}$$

where B_t is the biomass in year t and C_t is the total catch in year t . It is known that there are 3 types of production model listed below (Schaefer type, Fox type, and Pella Tomlinson type).

$$g(B) = rB \left(1 - \frac{B}{K}\right) \tag{2}$$

$$g(B) = rB \left(\log \frac{K}{B}\right) \tag{3}$$

$$g(B) = rB \left(1 - \left(\frac{B}{K}\right)^z\right) \tag{4}$$

Here, r is the intrinsic growth rate, K is the carrying capacity, and z denotes the shape parameter of Pella-Tomlinson model which control its shape and MSY level. These three types of functions were used in our analysis, though z is fixed to 0.5 to adjust MSY level to the midway between those of Schaefer type and Fox type. Note that MSY level of these 3 are 50%, 44.4%, 36.8% of carrying capacity, respectively (see Fig 3).

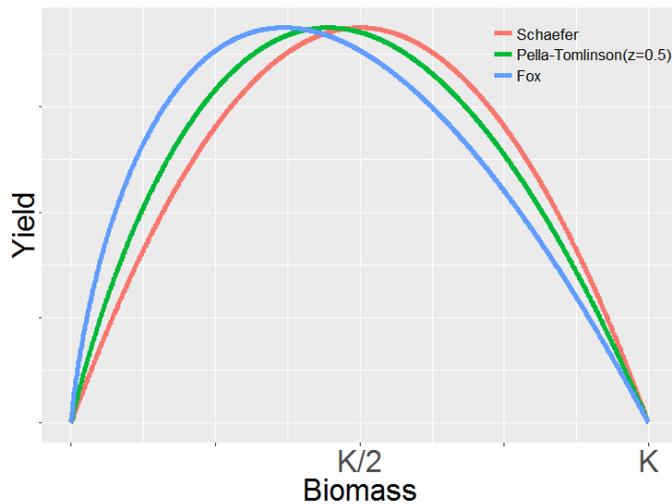


Figure 3. Relationship between biomass and yields of 3 types of production models used in this paper.

Table 1. MSY and MSY level for production models

Model	MSY	MSY level
Schaefer	$\frac{rK}{4}$	$\frac{K}{2}$
Fox	$\frac{rK}{e}$	$\frac{K}{e}$
Pella-Tomlinson	$rKz(z+1)^{-1-\frac{1}{z}}$	$K\left(\frac{1}{z+1}\right)^{\frac{1}{z}}$

The observed data are represented by vectors with value for yields and CPUE denoted by C_t and I_t , respectively, where $t=1960, \dots, 2015$ is the index for the year. Since annual catch in 1950 to 1959 is very little, we assumed that biomass in 1960 reached to carrying capacity. The relationship between CPUE and biomass is:

$$I_t = qB_t \quad (5)$$

where q is the catchability coefficient. And as model's error structure, we employ a state-space model that can be assumed existence of both observation error and process error as follows;

$$\begin{aligned} B_{t+1} &= (B_t + g(B_t) - C_t)e^{\varepsilon_t} \\ I_t &= qB_t e^{\eta_t} \\ \varepsilon_t &\sim N(0, \sigma_\varepsilon^2) \\ \eta_t &\sim N(0, \sigma_\eta^2) \end{aligned} \quad (6)$$

where ε_t and η_t are respectively the process and observation errors in year t . Since there is a hierarchical structure in this model, it needs to be used Bayesian estimation to get information about each parameter. A statistical estimate can be obtained by using these Bayes posterior probability distributions from MCMC sampling. We used three types of Bayesian state-space production models and fitted to CPUE series by MCMC. Furthermore, as joint CPUE were combined form of Japanese STD CPUE (1960-2015) and Taiwanese STD CPUE (1979-2015), we constructed another model that have different catchability coefficient between before 1978 and after 1979 as below.

$$\begin{aligned} I_{1960, \dots, 1978} &= q_1 B_{1960, \dots, 1978} \\ I_{1979, \dots, 2015} &= q_2 B_{1979, \dots, 2015} \end{aligned} \quad (7)$$

So when Joint CPUE data are used, numbers of modes to compare are six. We tried to find the best model from each case with using 2 CPUE series.

3.3 MCMC sampling

In this paper, a MCMC method (Gibbs sampling) was used to estimate parameters of models and posterior distributions of benchmarks (MSY , F_{msy} , B_{msy} , F -ratio, B -ratio). To simplify the calculation by MCMC, we input rescaled catch data which was divided by 100000. Accordingly, carrying capacity, catchability coefficient and MSY were also reparametrized as below.

$$\begin{aligned} \hat{K} &= \frac{K}{100000} \\ \hat{q} &= 100000 * q \\ \hat{MSY} &= \frac{MSY}{100000} \end{aligned} \quad (8)$$

And the state-space model used here is reparametrized by the calculation of the proportion of the annual biomass in relation to the carrying capacity (Depletion level), which results in improvement in the performance of MCMC sampling.

$$\begin{aligned} D_{t+1} &= D_t + g(D_t) - \frac{C_t}{\hat{K}} \\ I_t &= \hat{q}\hat{K}D_t \end{aligned} \quad (9)$$

As non-informative prior distribution, a uniform distribution was used for all the parameters in the models ($r, \hat{K}, \hat{q}, \sigma_\varepsilon, \sigma_\eta$). The range of prior distribution was set by the trial and error process. Checking the result of posterior distribution of each parameter, we adjusted the range of prior distribution to slightly wider than posterior distribution. As estimated values of each parameter, we employed posterior medians.

3.4 Model selection by predictive evaluation

We separated the CPUE data into two parts by years. Using the former part, we estimated the values of parameters of each model to fit. And using the latter part of catch data, we predicted CPUE corresponded to the latter part by model. If predicted CPUE fit CPUE data well, we can assume that the model has good predictive ability. Due to the way to separate the data, we made 10 cases in this research. Residuals between logarithm of predicted CPUE and CPUE data were calculated in every case (see Table 2), and square sums were used as an indexes of predictive abilities of models. Then we compared the indexes of each model in the 10 cases and attempted to select the best one.

Table 2. 10 Cases of separating data.

	Using years for parameter estimation	Prediction years
Prediction for 5 years	1960-2010 (51years)	2011-2015
	1960-2005 (46years)	2006-2010
	1960-2000 (41years)	2001-2005
	1960-1995 (36years)	1996-2000
Prediction for 10 years	1960-2005 (46years)	2006-2015
	1960-2000 (41years)	2001-2010
	1960-1995 (36years)	1996-2005
Prediction for 15 years	1960-2000 (41years)	2001-2015
	1960-1995 (36years)	1996-2010
Prediction for 20 years	1960-1995 (36years)	1996-2015

3.5 Assessments and risk assessments

We assessed the status of the bigeye tuna stock in the Indian Ocean by selected model. In particular, we estimated several assessment benchmarks ($MSY, F_{msy}, B_{msy}, Fratio, Bratio$, depletion level) and created Kobe plot. With that, the probability that the latest plot (2016) is distributed each of 4 zone of Kobe plot and the one that biomass in 2016 is less than MSY level were calculated. Moreover, future risks were evaluated after 3 years (2019) and 10 years (2026) from 2016 by 5 different scenarios listed below. As we couldn't control, catch amount in 2016 were assumed the mean of 2013-2015 catch (98,149t). Scenario 3 is a case that we assumed fish stock would be harvested by the mean catch in every year constantly. And in other scenarios, catch level was assumed to be $\pm 20\%$ and $\pm 40\%$ from scenario 3.

Table 3. Scenarios for risk assessment.

	Fishing Pressure	Catch amount in each year
Scenario 1	-40%	58889t
Scenario 2	-20%	78519t
Scenario 3	0%	98149t
Scenario 4	20%	117778t
Scenario 5	40%	137408t

4. Results and Discussion

4.1 Predictive Evaluation

From the verification predictive abilities, there is no distinctive difference between models when we predict CPUE of less than 10 years (see Table 4). But it seems that longer period of prediction which is hardly made in practice tends to make notable differences of values. By comparison residual sums, Pella-Tomlinson model for Japanese CPUE and Schaefer model with assumption of changing catchability coefficient for Joint CPUE(Tropical) can follow the behavior of CPUE the most in exceeding 15 years of prediction. We selected Schaefer model with assumption of change in catchability coefficient using Joint CPUE(Tropical) for base case.

Table 4. Residual sum of squares between predicted and observed CPUE in log-spaceo. (The minimum numbers of the 3 models in each case were highlighted in red).

(1). Japanese STD CPUE(Tropical)

		From 1996	From 2001	From 2006	From 2011
Schaefer	Prediction for 5 years	0.2879409	0.1489222	0.1123339	0.1082243
Pella		0.2475226	0.2085962	0.091482	0.1107518
Fox		0.2994303	0.2341863	0.0977125	0.1026189
Schaefer	Prediction for 10 years	1.062124	0.2682026	0.2048133	
Pella		0.9631629	0.3078535	0.2184824	
Fox		1.176578	0.3710539	0.1592666	
Schaefer	Prediction for 15 years	1.489356	0.6369986		
Pella		1.336354	0.3734999		
Fox		1.69553	0.3933857		
Schaefer	Prediction for 20 years	1.503132			
Pella		1.351571			
Fox		1.735366			

(2). Joint CPUE (Tropical)

		From 1996	From 2001	From 2006	From 2011
Schaefer		0.2021467	0.1359474	0.04551028	0.3117213
Pella		0.1795932	0.1508725	0.02726486	0.3023951
Fox	Prediction for	0.2013443	0.2339122	0.02727753	0.3975763
Schaefer (q_i)	5 years	0.05205223	0.07819224	0.08343136	0.276779
Pella (q_i)		0.1031325	0.09861153	0.04917825	0.3656438
Fox (q_i)		0.06717824	0.1548699	0.02613115	0.3618454
Schaefer		1.053117	0.1575708	0.3062122	
Pella		0.9615204	0.1860986	0.328444	
Fox	Prediction for	1.051109	0.3527293	0.3551507	
Schaefer (q_i)	10 years	0.2140069	0.1268748	0.3581645	
Pella (q_i)		0.2290101	0.1173415	0.3196302	
Fox (q_i)		0.3282598	0.1956832	0.3792547	
Schaefer		1.789409	0.4845242		
Pella		1.640563	0.6014716		
Fox	Prediction for	1.793014	0.9404399		
Schaefer (q_i)	15 years	0.2786102	0.3936652		
Pella (q_i)		0.5017086	0.4564581		
Fox (q_i)		0.4950874	0.6873875		
Schaefer		3.084629			
Pella		2.877596			
Fox	Prediction for	3.131945			
Schaefer (q_i)	20 years	0.6925678			
Pella (q_i)		0.8125204			
Fox (q_i)		1.191292			

4.2 Assessments

Compare to the previous assessment results by SS3, ASPIC and ASPM, outcomes in this paper suggested stock of bigeye tuna in Indian Ocean might be more pessimistic because the stock status was estimated to be in the red zone of Kobe plot (see figure 4). This result is thought to be derived from latest Joint CPUE's decreasing trend.

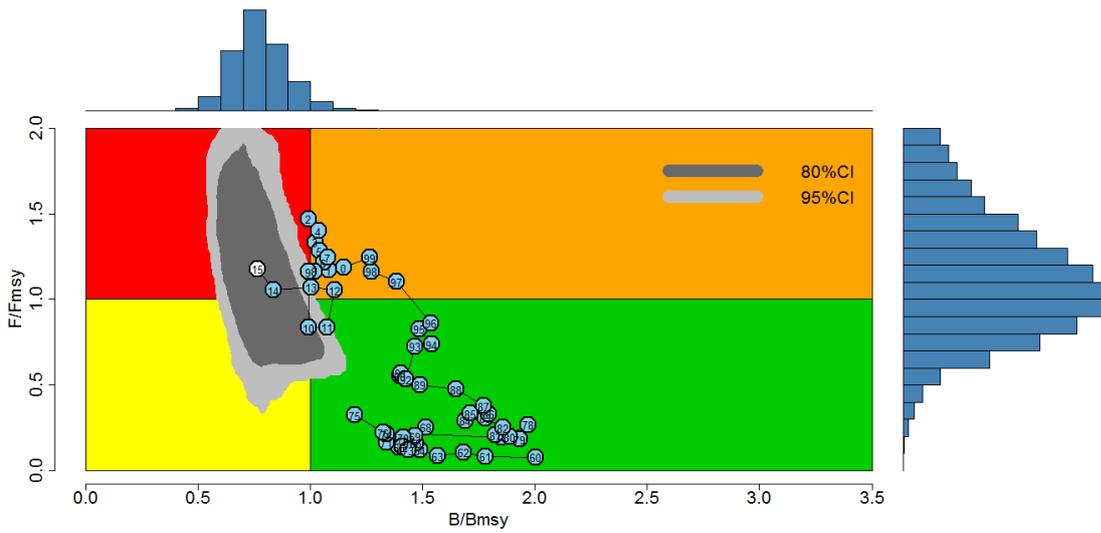


Figure 4. Kobe plot (white plot represents 2015) and histogram of the plot.

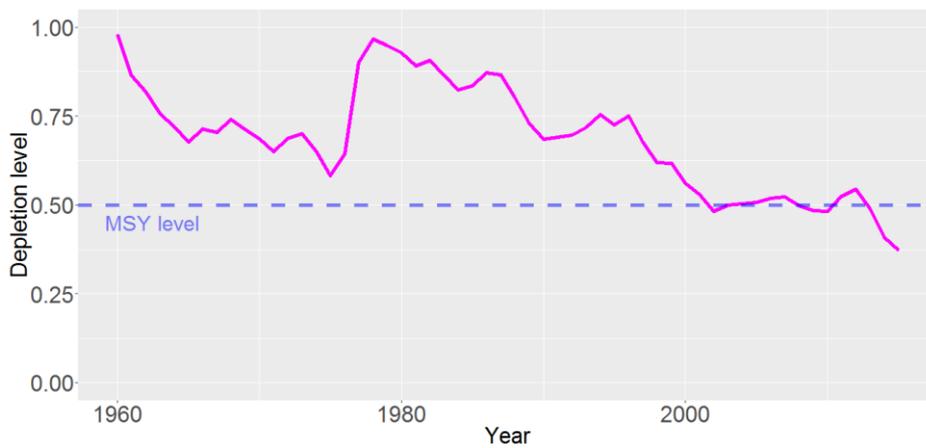


Figure 5. Trajectory of depletion level.

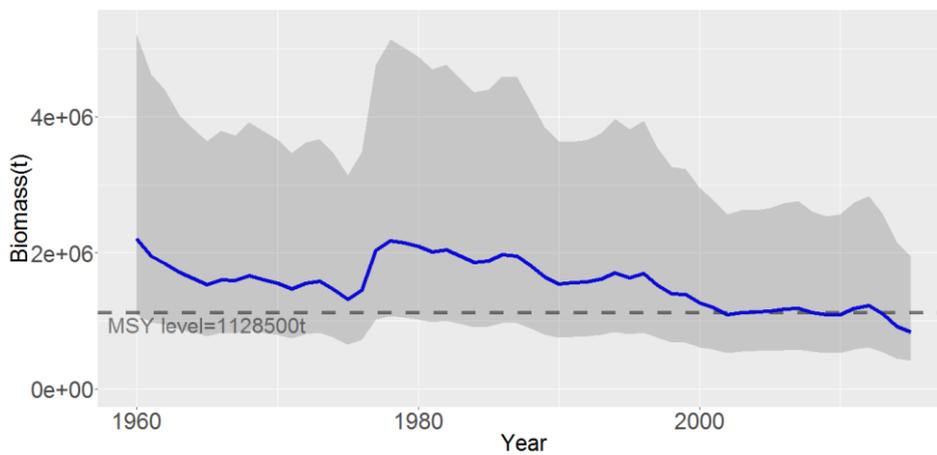


Figure 6. Trajectory of biomass(t) and 80% credible interval.

Table 5. Indian Ocean bigeye tuna stock status summary based on the Bayesian state-space Schaefer model with assumption of change in q .

Management Quantity	Estimate
Most recent catch estimate (t) (2015)	92,736
Mean catch over last 5 years (t) (2011-2015)	101,515
MSY (1000 t) (80%CI)	104.95 (57.1-147.9)
F(Current)/F(MSY) (2015) (80% CI)	1.17 (0.63-1.69)
B(Current)/B(MSY) (2015) (80% CI)	0.76 (0.74-2.3)

4.3 Risk assessment

We made future projections of B-ratio (Figure 7) and F-ratio (Figure 8) and probability table (Table 6). We got a pessimistic result for risk assessment of status of stock from previous works. The result suggested annual catch amount should be reduced at least 20%.

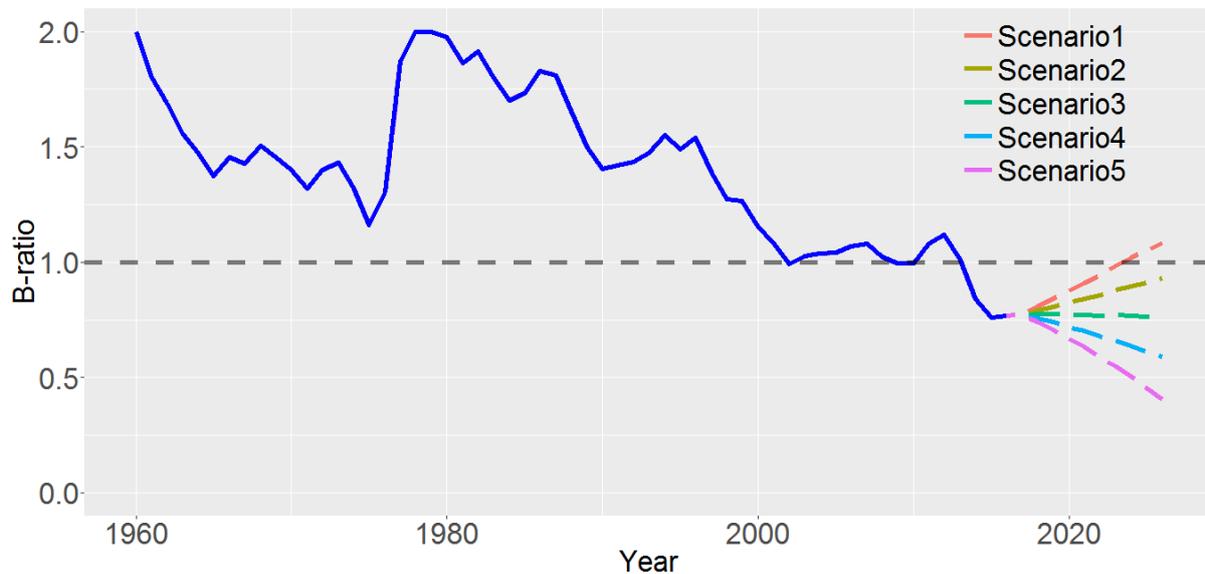


Figure 7. Future projection of B-ratio in each scenario.

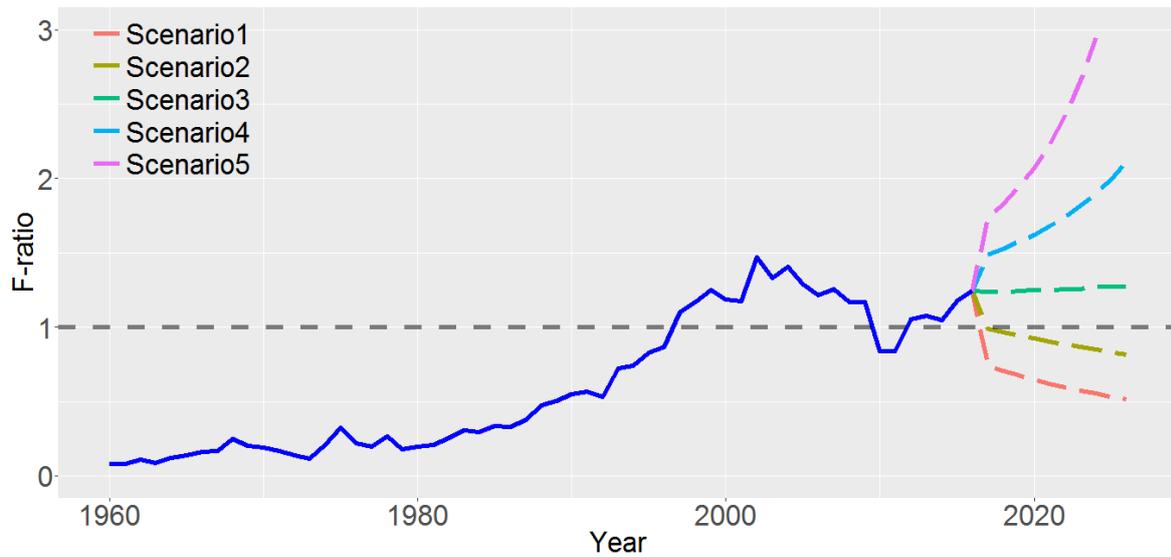


Figure 8. Future projection of F-ratio in each scenario.

Table 6. The probability that the plots (2016,2019,2026) are distributed in each 4 zone of Kobe plot and the probability that B and F in 2015 may be less than MSY level.

Year	Red	Orange	Yellow	Green	$B < B_{msy}$	$F < F_{msy}$	
2016	67.33%	1.42%	22.74%	8.59%	64.71%	37.65%	
Scenario1	27.41%	0.64%	41.65%	30.35%	60.57%	74.21%	
Scenario2	45.36%	1.2%	27.7%	25.83%	62.42%	53.51%	
2019	Scenario3	61.55%	2.24%	15.3%	21.01%	64.01%	38.23%
Scenario4	72.91%	4.22%	7.44%	15.48%	65.55%	28.67%	
Scenario5	79.51%	6.65%	3.78%	10.13%	66.97%	21.69%	
Scenario1	24.36%	0.76%	19.97%	54.95%	48.51%	82.36%	
Scenario2	40.54%	1.41%	14.54%	43.52%	56.71%	58.75%	
2026	Scenario3	57.19%	2.31%	8.69%	31.89%	64.08%	38%
Scenario4	71.81%	3.19%	4.56%	20.49%	69.97%	25.55%	
Scenario5	81.89%	3.97%	2.39%	11.79%	74.64%	17.94%	

5. Further discussion and future works

We applied predictive evaluation method to bigeye tuna catch statistics for the first attempt. Since it seems that predictive abilities have differences between models clearly in prediction of more than 15 years, the results indicated application potentiality of the Model selection using evaluation predictive abilities of Models. But predictive evaluation method depends on CPUE data greatly, therefore it

might be said that CPUE have to strongly reflect true biomass. For the further analysis, predictive evaluation method must be inspected its estimation accuracy by simulation data and should be more sophisticated in the future. In addition, we should look for another criterion for predictive evaluation and try to apply to other complex models.

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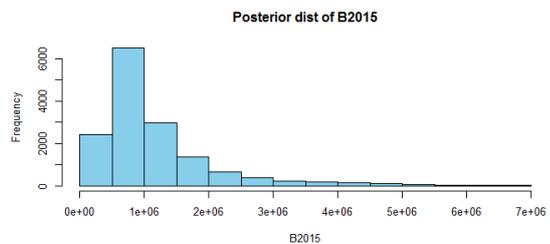
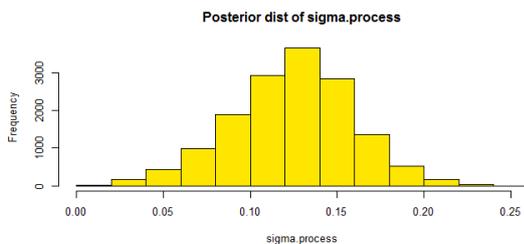
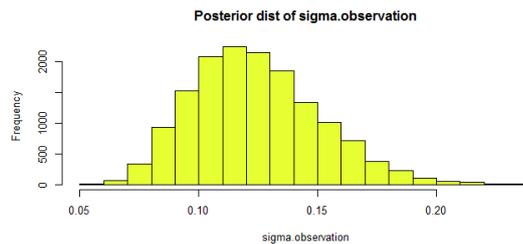
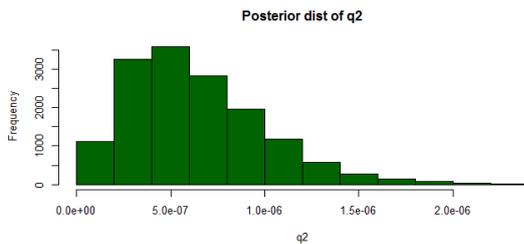
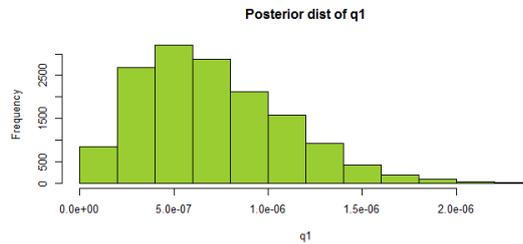
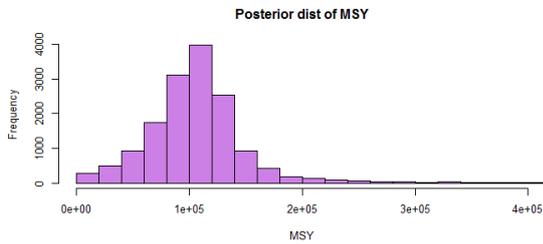
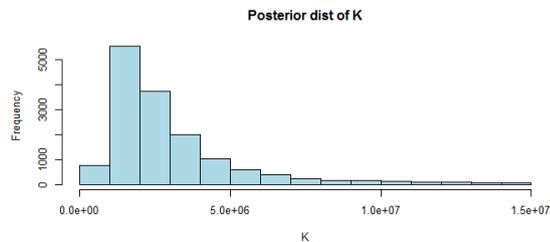
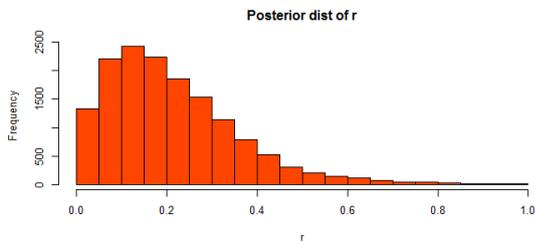
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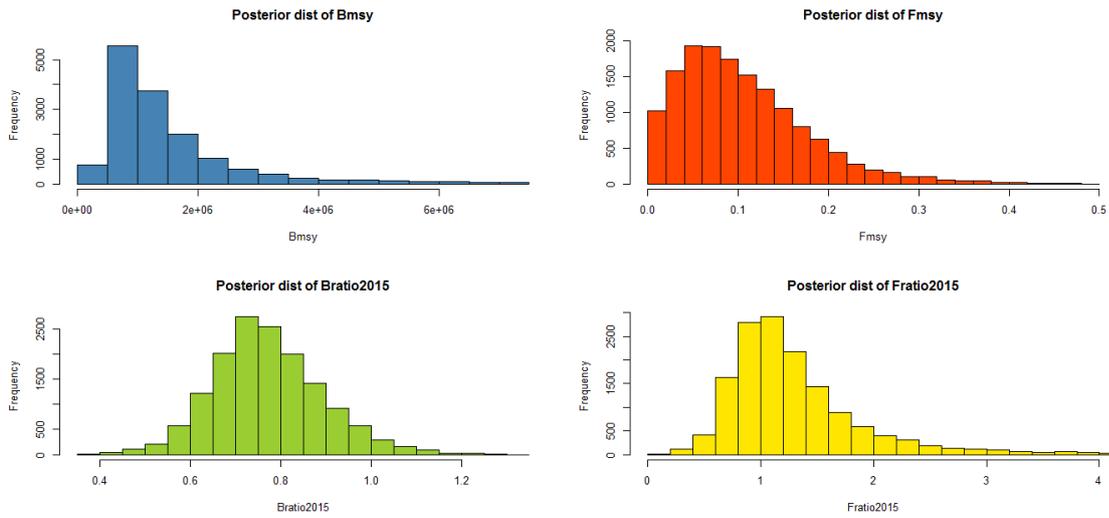
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Appendix1:

Prior and posterior distribution of each parameter by Bayesian state-space Schaefer model with chang in q

	Prior
r	U(0,1)
K	U(300000,1500000)
q_1	U(0,0.000025)
q_2	U(0,0.000023)
σ_η	U(0,0.24)
σ_ε	U(0,0.25)





Appendix2:

Estimated value of each parameter and estimated values by Bayesian state-space Schaefer model with chang in q

	Mean	Median	SD	2.5%	97.5%
r	0.211	0.183	0.144	0.017	0.574
$K(1000t)$	2999	2257	2316.7	863.2	10420
$q_1(10^{-7})$	7.06	6.51	3.78	1.45	15.6
$q_2(10^{-7})$	6.36	5.74	3.58	1.28	15
σ_η	0.124	0.122	0.027	0.08	0.182
σ_ε	0.124	0.126	0.035	0.051	0.191