# Update of Age and sex specific Natural mortality of the blue shark (Prionace glauca) in the North Pacific Ocean 

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## Introduction

In the last stock assessment of blue shark (Prionace glauca) in the North Pacific, age and sex-specific natural mortality (indicated as " $M$ " hereafter) of this stock was used as the input data for Stock Synthesis model (Rice et al. 2014). In Rice and Semba (2014), the estimator by Peterson and Wroblewski (1984) and Chen and Watanabe (1986) and the growth equation by Nakano (1994) and Hsu et al. (2011) were used. The estimator of M applied is divided into weight-based (Peterson and Wroblewski 1984) and age-based (Chen and Watanabe 1986). The age and sex-specific $M$ based on the former estimator was adopted in the stock assessment with the assumption of maximum age of 30 . However, in the calculation by Rice and Semba (2014), the coefficient assigned for the dry weight (1.92) was mistakenly applied to wet weight of North Pacific blue shark, instead of that for the wet weight (1.28), which needs to be corrected in the upcoming stock assessment.

While the weight-based estimator has been widely applied in the stock assessment of fishery resources, the effect of variance of weight among individual is suggested to be large compared to that of body length. For example, large difference in weight would be expected between pregnant female and adult male with same age. In this context, we show the age and sex-specific $M$ based on the length-based method used in the stock assessment of Atlantic yellowfin tuna (Thunnus albacares) this year (Method 2 in Walter et al. 2016) as well as the modified M by Peterson and Wroblewski (1984). In addition, we calculated the age and sexspecific M based on another estimators included in the review by Kenchington (2014) for comparison. Regarding growth curve necessary for the estimation of M, von Bertalanffy growth curve (VBGC) by Nakano (1994) and Hsu et al. (2011) are used for the sake of consistency with the past estimates. In addition, the estimates based on VBGC by Fujinami et al. (2016) is also indicated for reference.

## Methods

## Basic equation

We show several equations necessary to estimate of natural mortality-at-age, below (i.e. growth curve, weight-length relationship, conversion equation between total length (TL) and precaudal length (PCL)). The parameter of VBGC (eq. 1) for each sex from Nakano (1994), Hsu et al. (2011), and Fujinami et al. (2016) is indicated below.

## Growth curve (von Bertalanffy (1938))

$$
\begin{equation*}
L_{t}(c m)=L_{\infty}\left(1-\exp \left(-K\left(t-t_{0}\right)\right)\right. \tag{1}
\end{equation*}
$$

1. Nakano (1994)

$$
\begin{aligned}
& P C L: L_{\infty}=289.7, K=0.129, t_{0}=-0.756 \quad \text { Male } \\
& P C L: L_{\infty}=243.3, K=0.144, t_{0}=-0.849 \quad \text { Female }
\end{aligned}
$$

2. Hsu et al. (2011)

$$
\begin{aligned}
& T L: L_{\infty}=375.8, K=0.121, t_{0}=-1.554 \text { Male } \\
& T L: L_{\infty}=317.4, K=0.172, t_{0}=-1.123 \text { Female }
\end{aligned}
$$

3. Fujinami et al. (2016)

$$
\begin{aligned}
& \text { PCL: } L_{\infty}=284.8, K=0.117, t_{0}=-1.34 \text { Male } \\
& \text { PCL: } L_{\infty}=256.3, K=0.147, t_{0}=-0.97 \text { Female }
\end{aligned}
$$

,where $L_{t}, L_{\infty}, K$ and $t_{0}$ denote body length at age $t$, asymptotic length, von Bertalanffy growth coefficient and theoretical age at which the organism was 0 length, respectively.

Body length used Nakano (1994) and Fujinami et al. (2016) is PCL, while that used in Hsu et al. (2011) is TL. For unifying the body length, TL and PCL was converted using the conversion equation (eq. $2 \mathrm{a}, \mathrm{b}$ ) below;

Length (PCL)-length (TL) (Nakano 1985)

$$
\begin{equation*}
P C L(\mathrm{~cm})=0.762 * T L(\mathrm{~cm})-0.2505 \tag{2a}
\end{equation*}
$$

$$
\begin{equation*}
T L(\mathrm{~cm})=(P C L+0.2505) / 0.762 \tag{2b}
\end{equation*}
$$

Sex-specific conversion equation between wet body weight (kg) and PCL (eq. 3) was cited from Nakano (1994) as follows;

## Wet weight (kg)-body length(PCL) relationship (Nakano 1994)

$$
\begin{gather*}
W(\mathrm{~kg})=5.388 * 10^{-6} P C L^{3.102}(\mathrm{~cm}) \text { Male }  \tag{3a}\\
W(\mathrm{~kg})=3.293 * 10^{-6} P C L^{3.225}(\mathrm{~cm}) \text { Female } \tag{3b}
\end{gather*}
$$

,where W is body weight.

## Mortality estimator

As empirical and theoretical estimator for age specific $M$ commonly used, we focused on the method by Lorenzen (1996), Gislason et al. (2010) and Charnov et al. (2013) as well as that by Peterson and Wroblewski (1984) and Chen and Watanabe (1989). Regarding Peterson and Wroblewski (1984), the previous study used the dry weight (eq. 4), but this study used the wet weight (eq. 5).

1. Peterson and Wroblewski (1984)

$$
\begin{align*}
& M\left(\text { year }^{-1}\right)=1.92 * W^{-0.25}(g) \text { Dry weights }  \tag{4}\\
& M\left(\text { year }^{-1}\right)=1.28 * W^{-0.25}(g) \text { Wet weights } \tag{5}
\end{align*}
$$

2. Chen and Watanabe (1989)

$$
\begin{align*}
& M\left(\text { year }^{-1}\right) \\
& =\left\{\begin{array}{cc}
\frac{K}{1-e^{-K\left(t-t_{0}\right)}} \quad\left(t \leq t_{M}\right) \\
\frac{K}{1-e^{-K\left(t_{M}-t_{0}\right)}+\left(t-t_{M}\right) K e^{-K\left(t_{M}-t_{0}\right)}-0.5 *\left(t-t_{M}\right)^{2} K^{2} e^{-K\left(t_{M}-t_{0}\right)}} & \left(t \geq t_{M}\right)
\end{array}\right. \tag{6}
\end{align*}
$$

$$
t_{M}=-\frac{1}{K} \log \left|1-e^{k t_{0}}\right|+t_{0}
$$

, where $t_{M}$ is age at end of reproductive span, i.e. age at the intersection of the stable and senescent growth phases.
3. Lorenzen (1996)

$$
\begin{equation*}
M\left(y e a r^{-1}\right)=3.00 * W^{-0.288}(g) \tag{7}
\end{equation*}
$$

4. Gislason et al. (2010)

$$
\begin{equation*}
M\left(\text { year }^{-1}\right)=1.73 * T L^{-1.61}(\mathrm{~cm}) * L_{\infty}^{1.44} * K \tag{8}
\end{equation*}
$$

5. Charnov et al. (2012)

$$
\begin{equation*}
M\left(\text { year }^{-1}\right)=\left(T L(\mathrm{~cm}) / L_{\infty}\right)^{-1.5} * K \tag{9}
\end{equation*}
$$

6. Method 2 in Walter et al. (2016)

$$
\begin{equation*}
M\left(\text { year }^{-1}\right)=\frac{M_{T}\left(t_{\max }-t_{c}\right)}{\ln \left(\frac{l_{c}}{l_{c}+L_{\infty}\left(\exp \left(K\left(t_{\max }-t_{c}\right)\right)-1\right)}\right)} \ln \left(\frac{l_{t}}{l_{t}+L_{\infty}(\exp (K)-1)}\right) \tag{10}
\end{equation*}
$$

, where $M_{T}$ is Target M defined as the M as obtained from an external study (Walter et al. 2016) and $t_{c}$ is the age at first full recruitment, $l_{c}$ is the body length at $t_{c}, l_{t}$ is body length at age t , and $t_{\max }$ is the maximum age. As the base case here, we set Target M as 0.23 (Campana et al. 2004), $t_{c}$ as 0 (discussed later), $t_{\max }$ as 30 (Rice and Semba 2014, Rice et al. 2014). The concept of this method and derivation of equation (10) is described in Appendix 1.

Throughout all estimators, growth parameter $\left(L_{\infty}, K, t_{0}\right)$ were used from equation by Nakano (1994), Hsu et al. (2011), and Fujinami et al. (2016) indicated above.

Given the uncertainty associated with the interpretation of $t_{c}$, age and sex-specific M by six different $t_{c}(0 \sim 5)$ was calculated for the growth curve by Nakano (1994) and Fujinami et al. (2016).

In addition, to check the effect of $t_{\max }$, we also calculated age and sex-specific M by (1) six different $t_{c}(0 \sim 5)$ assuming $t_{\max }=20$ for the growth curve by Nakano (1994) and (2) three

## Results

Sex-specific growth curve based on Nakano (1994), Hsu et al. (2011) and Fujinami et al. (2016) indicated slight difference in growth rate between studies for female but almost same growth rate for male (Figure 1, Table 1).

The revised M based on Peterson and Wroblewski (1984) was lower than the past estimates in both sex and growth curve. This is reasonable considering that the corrected coefficient (1.28) is smaller than that used in the past report (1.92).

Estimates of age and sex-specific M based on growth curve by Nakano (1994) were indicated in Table 2 and Figure2. In both sexes, M at age 0 is the highest in the estimates based on Charnov et al. (2012) and the second highest in that by Gislason et al. (2010). In general, M by length-based estimators were higher than those by weight-based estimators, while M by Chen and Watanabe (1989) and Walter et al. (2016) were intermediate, especially before M reaches at plateau. M at age 0 declined most steeply from a high of $0.37 \sim 4.50$ (male) and $0.36 \sim 3.65$ (female) to approximately $0.20 \sim 1.40$ (male) and $0.21 \sim 1.27$ (female) at age 1 . Thereafter M declined gradually for both sex by age at maturity to $0.10 \sim 0.34$ (age 5 for male) and 0.09~0.29 (age 6 for female). Subsequently, M showed slow decrease over lifetime of the fish in almost all estimators and below 0.2 for both sexes after age 10

Estimates of age and sex-specific M based on growth curve by Hsu et al. (2011) were indicated in Table 3 and Figure3. As with the previous result, M at age 0 is the highest in the estimates based on Charnov et al. (2012) and the second highest in that by Gislason et al. (2010) in both sexes, but the absolute value was smaller than those based on Nakano (1994)'s growth curve. Relative relationship between length-based and weight-based estimates were similar with the case in the growth curve by Nakano (1994). M at age 0 declined most steeply from a high of
$0.23 \sim 1.70$ (male) and $0.26 \sim 2.34$ (female) to approximately $0.16 \sim 0.88$ (male) and $0.17 \sim 1.02$ (female) at age 1 . Thereafter M declined gradually for both sex by age at maturity to $0.09 \sim 0.30$ (age 5 for male) and 0.08~0.29 (age 6 for female). Subsequently, male M showed slow decrease and stabilized around 0.1 after age 20 in all estimators. Female $M$ showed similar trend with males but estimates based on Chen and Watanabe (1989) showed increasing trend from 0.20 to 0.55 after age 20 .

The effect of $t_{c}$ in the method by Walter et al. (2016) was shown in Figure 4 and calculated value is indicated in Appendix 2. M with assumption of $t_{c}=0$ is the lowest and that of $t_{c}=5$ is the highest of all estimate in both sexes. Especially, estimates with assumption of $t_{c}=4$ and $t_{c}=5$ was closely similar, while difference of $t_{c}$ between 0 and 2 was moderate in both sexes. If $t_{\max }$ changed from 30 to $20, \mathrm{M}$ got smaller in every $t_{c}$ for both sexes (Figure 5). The estimates for three different $t_{\max }$ for 3 VBGC with $t_{c}=0$ is shown in Appendix 3.

## Discussion

Regardless of sex and growth curve used, estimates by Charnov et al. (2012) and Gislason et al. (2010) were much higher than other estimates. These are length-based estimates and also based on the regression between empirical value of $M$ and length from variety of organism (e.g., sand eel, herring, and seahorse). Application of empirical value from taxa totally different from shark might affect this high M observed in early ages. Although empirical information on M at early ages in blue shark is lacking, M at age 0 by Charnov et al. (2012) and Gislason et al. (2010), indicating 1.7~4.5 in the former and 1.3~3.7 in the latter, would be unrealistic, given the empirical estimate of M at age 0 for bluefin tuna (1.6), yellowfin tuna (lower than 0.8 except for M in 21-30 cm folk length) and bigeye tuna ( $0.15-0.9$ for size classes $>40 \mathrm{~cm}$ folk length) (Hampton 2000, Mangel et al. 2010). Although M in later age was low and similar among estimators, unrealistically high $M$ at earlier age might influence the result of stock assessment.

The moderate M observed in estimator of Chen and Watanabe (1989) and Walter et al. (2016) might because they are length-based method with theoretical approach (not affected by empirical value). Regarding Chen and Watanabe (1989), the high value obtained from after age 20 in female for Hsu et al. (2011) was suggested to be unlikely and disregarded in Rice and Semba (2014). Although increase of $M$ due to senescence has been widely discussed, the degree of increase indicated here contain large uncertainly because long lived elasmobranchs are expected to have a relatively low M , particularly once they reach larger sizes (Rice and Semba 2014). Estimates by Walter et al. (2016) ranges from 0.7 and 1.0 depending on the growth curve used and do not show increasing trend observed in Chen and Watanabe (1989). Thus, within length-based estimator, method by Walter et al. (2016) may provide reasonable estimates of M.

Weight-based estimator provided relatively low M and revised estimates from Peterson and Wroblewski (1984) was the lowest of all estimates. Mean M across ages (i.e., constant M) from revised Peterson and Wroblewski (1984) and Lorenzen (1996) is 0.08-0.09 for Nakano (1994) and 0.08 for Hsu et al. (2011) in the former estimator and 0.14 for Nakano (1994) and 0.13 for Hsu et al. (2011) in the latter estimator, respectively. This is lower than constant M estimated for Lamna nasus (0.18, Aasen 1963), which seems unlikely given the life history parameter and fecundity of L. nasus. Given that estimator by Peterson and Wroblewski (1984) also relies upon the empirical value of M and weight for a wide variety of animals including chaetognatha and larval fish and most data consists of dry weight lighter than 1 kg , the application of this estimator might increase uncertainty on the M of blue shark.

In application of Walter's method, we set $M_{T}$ as 0.23 and $t_{c}$ as 0 . Regarding $M_{T}$, we regarded that constant M from meta-analysis by Campana et al. (2004) would be reasonable because it covers various parameter of this species and several estimators of M. Although we did not check the target age/size for which each estimator in this meta-analysis covered, we selected $t_{c}=0$, given the situation that constant M has been applied to all ages in the stock assessment in practice. If the estimator included in this meta-analysis only cover adult or juvenile, our assumption (i.e., $t_{c}=0$ ) may affect the results. Considering the room for discussion
on the value of $t_{c}$, arising from the original definition in Walter et al. (2016), we showed the M on various $t_{c}(0 \sim 5)$ in the appendix 2 and 4 . Although the effect of $t_{c}$ and $t_{\max }$ was not so large, the assumption of $M_{T}$ is suggested to have impacts on the results.

In this document, we reviewed the previous estimator of age and sex-specific $M$ and compared the estimates based on length- and weight- based estimators. The estimator based on regression analysis for empirical value would cause over or under estimation for M when applied to group which is not covered in the original data. Given the discussion above, we suggest that age and sex-specific $M$ based on method2 in Walter et al. (2016) is worth to be discussed when inputted into stock synthesis modelling. Regarding the growth curve to be used for the estimation, it is suggested that Fujinami et al. (2016) equation provided plausible estimates throughout the life span of the North Pacific blue shark, when applied to Walter's method with $t_{c}=0$. As there are room for discussion over both estimator and parameter such as $t_{c}$ and $t_{\max }$, we hope this document is used as the tool for further discussion for the upcoming stock assessment of this stock.

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Table 1. Length at age by sex for growth curve by Nakano (1994), Hsu et al.(2011), and Fujinami et al. (2016). Body length is converted into PCL.

| Age | $\begin{gathered} \text { Nakano } \\ \text { (1994) Male } \end{gathered}$ | Nakano (1994) Female | Hsu et al. (2011) Male | Hsu et al. (2011) Female | Fujinami et al. (2016) Male | Fujinami et al. (2016) Female |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 26.9 | 28.0 | 48.8 | 42.2 | 41.3 | 34.1 |
| 1 | 58.7 | 56.9 | 75.9 | 73.7 | 68.2 | 64.4 |
| 2 | 86.7 | 81.9 | 99.8 | 100.3 | 92.1 | 90.7 |
| 3 | 111.2 | 103.5 | 121.1 | 122.6 | 113.4 | 113.3 |
| 4 | 132.8 | 122.3 | 139.9 | 141.4 | 132.3 | 132.9 |
| 5 | 151.8 | 138.5 | 156.5 | 157.2 | 149.2 | 149.7 |
| 6 | 168.5 | 152.6 | 171.3 | 170.6 | 164.1 | 164.3 |
| 7 | 183.2 | 164.7 | 184.4 | 181.8 | 177.5 | 176.9 |
| 8 | 196.1 | 175.3 | 196.0 | 191.2 | 189.3 | 187.7 |
| 9 | 207.4 | 184.4 | 206.3 | 199.2 | 199.9 | 197.1 |
| 10 | 217.4 | 192.3 | 215.4 | 205.9 | 209.2 | 205.2 |
| 11 | 226.1 | 199.1 | 223.4 | 211.5 | 217.6 | 212.2 |
| 12 | 233.8 | 205.1 | 230.6 | 216.3 | 225.0 | 218.2 |
| 13 | 240.6 | 210.2 | 236.9 | 220.3 | 231.6 | 223.4 |
| 14 | 246.5 | 214.6 | 242.5 | 223.7 | 237.5 | 227.9 |
| 15 | 251.7 | 218.5 | 247.5 | 226.5 | 242.7 | 231.8 |
| 16 | 256.3 | 221.8 | 251.9 | 228.9 | 247.3 | 235.1 |
| 17 | 260.4 | 224.7 | 255.8 | 230.9 | 251.5 | 238.0 |
| 18 | 263.9 | 227.2 | 259.2 | 232.6 | 255.2 | 240.5 |
| 19 | 267.0 | 229.3 | 262.3 | 234.0 | 258.4 | 242.7 |
| 20 | 269.8 | 231.2 | 265.0 | 235.2 | 261.3 | 244.6 |
| 21 | 272.2 | 232.8 | 267.4 | 236.2 | 263.9 | 246.2 |
| 22 | 274.3 | 234.2 | 269.5 | 237.1 | 266.2 | 247.5 |
| 23 | 276.2 | 235.5 | 271.4 | 237.8 | 268.3 | 248.7 |
| 24 | 277.8 | 236.5 | 273.1 | 238.4 | 270.1 | 249.8 |
| 25 | 279.3 | 237.4 | 274.6 | 238.9 | 271.7 | 250.7 |
| 26 | 280.5 | 238.2 | 275.9 | 239.3 | 273.2 | 251.4 |
| 27 | 281.6 | 238.9 | 277.1 | 239.7 | 274.5 | 252.1 |
| 28 | 282.6 | 239.5 | 278.1 | 240.0 | 275.6 | 252.7 |
| 29 | 283.5 | 240.0 | 279.0 | 240.2 | 276.6 | 253.2 |
| 30 | 284.2 | 240.4 | 279.8 | 240.5 | 277.5 | 253.6 |

Table 2. Natural mortality-at-age by sex for each estimator, based on the growth curve by Nakano (1994).

| Nakano (1994) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Male |  |  |  |  |  |  |  | Female |  |  |  |  |  |  |  |
| Age | Rev. Peterson \&Wroblewski (1984) | Gislason et al. (2010) | $\begin{gathered} \hline \text { Chen \& } \\ \text { Watanabe } \\ (1989) \end{gathered}$ | Lorenzen (1996) | Charnov et al. <br> (2012) | Previous (Peterson \& Wroblewski 1984) | Walter et al. <br> (2016) tc=0 | $\begin{array}{\|c\|} \hline \text { Fujinami et al. } \\ \text { (2016) male } \\ \text { (Walters:tc=0) } \\ \hline \end{array}$ | $\begin{gathered} \hline \text { Rev. Peterson } \\ \text { \&Wroblewski } \\ (1984) \end{gathered}$ | Gislason et al. <br> (2010) | Chen \& Watanabe (1989) | Lorenzen (1996) | Charnovet al. (2012) | Previous (Peterson \& Wroblewski 1984) | Walter et al. (2016) tc=0 | Fujinami et al. (2016) female (Walters:tc=0) |
| 0 | 0.3676 | 3.6766 | 1.3883 | 0.7127 | 4.4974 | 0.5514 | 1.0072 | 0.7877 | 0.3640 | 2.9986 | 1.2513 | 0.7046 | 3.6454 | 0.5348 | 0.9093 | 0.8436 |
| 1 | 0.2007 | 1.0558 | 0.6364 | 0.3550 | 1.4064 | 0.3011 | 0.5744 | 0.5321 | 0.2056 | 0.9651 | 0.6160 | 0.3648 | 1.2677 | 0.3087 | 0.5421 | 0.5252 |
| 2 | 0.1484 | 0.5653 | 0.4312 | 0.2507 | 0.7859 | 0.2226 | 0.4195 | 0.4139 | 0.1532 | 0.5379 | 0.4279 | 0.2601 | 0.7354 | 0.2327 | 0.4038 | 0.3977 |
| 3 | 0.1223 | 0.3786 | 0.3359 | 0.2006 | 0.5410 | 0.1835 | 0.3395 | 0.3458 | 0.1268 | 0.3691 | 0.3384 | 0.2092 | 0.5177 | 0.1940 | 0.3310 | 0.3292 |
| 4 | 0.1066 | 0.2847 | 0.2813 | 0.1712 | 0.4148 | 0.1599 | 0.2910 | 0.3017 | 0.1109 | 0.2825 | 0.2865 | 0.1792 | 0.4036 | 0.1705 | 0.2865 | 0.2866 |
| 5 | 0.0961 | 0.2297 | 0.2461 | 0.1520 | 0.3396 | 0.1441 | 0.2585 | 0.2710 | 0.1003 | 0.2312 | 0.2530 | 0.1596 | 0.3349 | 0.1548 | 0.2566 | 0.2579 |
| 6 | 0.0886 | 0.1943 | 0.2218 | 0.1384 | 0.2905 | 0.1329 | 0.2354 | 0.2486 | 0.0928 | 0.1980 | 0.2297 | 0.1459 | 0.2898 | 0.1436 | 0.2354 | 0.2373 |
| 7 | 0.0831 | 0.1699 | 0.2040 | 0.1285 | 0.2564 | 0.1246 | 0.2183 | 0.2315 | 0.0872 | 0.1750 | 0.2127 | 0.1359 | 0.2583 | 0.1353 | 0.2197 | 0.2221 |
| 8 | 0.0788 | 0.1523 | 0.1906 | 0.1209 | 0.2315 | 0.1182 | 0.2052 | 0.2182 | 0.0830 | 0.1584 | 0.1999 | 0.1283 | 0.2354 | 0.1290 | 0.2077 | 0.2104 |
| 9 | 0.0755 | 0.1391 | 0.1802 | 0.1150 | 0.2128 | 0.1132 | 0.1949 | 0.2075 | 0.0796 | 0.1460 | 0.1900 | 0.1224 | 0.2182 | 0.1240 | 0.1983 | 0.2013 |
| 10 | 0.0728 | 0.1290 | 0.1719 | 0.1103 | 0.1984 | 0.1091 | 0.1867 | 0.1989 | 0.0770 | 0.1364 | 0.1822 | 0.1177 | 0.2049 | 0.1200 | 0.1908 | 0.1940 |
| 11 | 0.0706 | 0.1211 | 0.1653 | 0.1065 | 0.1870 | 0.1058 | 0.1800 | 0.1919 | 0.0748 | 0.1290 | 0.1759 | 0.1139 | 0.1944 | 0.1168 | 0.1848 | 0.1882 |
| 12 | 0.0688 | 0.1147 | 0.1598 | 0.1033 | 0.1779 | 0.1031 | 0.1745 | 0.1860 | 0.0731 | 0.1230 | 0.1709 | 0.1109 | 0.1861 | 0.1142 | 0.1799 | 0.1834 |
| 13 | 0.0672 | 0.1096 | 0.1553 | 0.1007 | 0.1704 | 0.1009 | 0.1700 | 0.1810 | 0.0717 | 0.1183 | 0.1667 | 0.1083 | 0.1793 | 0.1120 | 0.1759 | 0.1795 |
| 14 | 0.0660 | 0.1054 | 0.1516 | 0.0985 | 0.1643 | 0.0990 | 0.1662 | 0.1768 | 0.0705 | 0.1143 | 0.1632 | 0.1063 | 0.1738 | 0.1102 | 0.1725 | 0.1762 |
| 15 | 0.0649 | 0.1019 | 0.1484 | 0.0967 | 0.1592 | 0.0974 | 0.1630 | 0.1733 | 0.0695 | 0.1111 | 0.1604 | 0.1045 | 0.1692 | 0.1087 | 0.1697 | 0.1735 |
| 16 | 0.0640 | 0.0990 | 0.1458 | 0.0952 | 0.1550 | 0.0960 | 0.1603 | 0.1702 | 0.0686 | 0.1085 | 0.1580 | 0.1031 | 0.1654 | 0.1074 | 0.1673 | 0.1712 |
| 17 | 0.0632 | 0.0965 | 0.1435 | 0.0938 | 0.1514 | 0.0949 | 0.1579 | 0.1676 | 0.0679 | 0.1062 | 0.1561 | 0.1018 | 0.1622 | 0.1064 | 0.1653 | 0.1693 |
| 18 | 0.0626 | 0.0944 | 0.1416 | 0.0927 | 0.1483 | 0.0939 | 0.1560 | 0.1653 | 0.0673 | 0.1044 | 0.1547 | 0.1008 | 0.1596 | 0.1055 | 0.1637 | 0.1676 |
| 19 | 0.0620 | 0.0927 | 0.1400 | 0.0918 | 0.1457 | 0.0930 | 0.1543 | 0.1634 | 0.0668 | 0.1028 | 0.1537 | 0.0999 | 0.1573 | 0.1047 | 0.1622 | 0.1663 |
| 20 | 0.0615 | 0.0911 | 0.1386 | 0.0909 | 0.1435 | 0.0923 | 0.1528 | 0.1617 | 0.0663 | 0.1014 | 0.1530 | 0.0992 | 0.1554 | 0.1040 | 0.1610 | 0.1651 |
| 21 | 0.0611 | 0.0898 | 0.1375 | 0.0902 | 0.1416 | 0.0917 | 0.1515 | 0.1602 | 0.0660 | 0.1003 | 0.1528 | 0.0985 | 0.1538 | 0.1035 | 0.1600 | 0.1641 |
| 22 | 0.0607 | 0.0887 | 0.1366 | 0.0896 | 0.1400 | 0.0911 | 0.1504 | 0.1589 | 0.0657 | 0.0993 | 0.1529 | 0.0980 | 0.1524 | 0.1030 | 0.1591 | 0.1632 |
| 23 | 0.0604 | 0.0878 | 0.1359 | 0.0890 | 0.1386 | 0.0906 | 0.1495 | 0.1577 | 0.0654 | 0.0985 | 0.1535 | 0.0975 | 0.1512 | 0.1026 | 0.1583 | 0.1625 |
| 24 | 0.0601 | 0.0869 | 0.1355 | 0.0886 | 0.1374 | 0.0902 | 0.1487 | 0.1567 | 0.0651 | 0.0978 | 0.1544 | 0.0971 | 0.1502 | 0.1022 | 0.1577 | 0.1619 |
| 25 | 0.0599 | 0.0862 | 0.1353 | 0.0882 | 0.1363 | 0.0899 | 0.1479 | 0.1559 | 0.0649 | 0.0972 | 0.1558 | 0.0968 | 0.1494 | 0.1019 | 0.1571 | 0.1613 |
| 26 | 0.0597 | 0.0856 | 0.1353 | 0.0878 | 0.1354 | 0.0895 | 0.1473 | 0.1551 | 0.0648 | 0.0967 | 0.1576 | 0.0965 | 0.1486 | 0.1017 | 0.1566 | 0.1609 |
| 27 | 0.0595 | 0.0851 | 0.1356 | 0.0875 | 0.1346 | 0.0893 | 0.1468 | 0.1544 | 0.0646 | 0.0962 | 0.1598 | 0.0962 | 0.1480 | 0.1014 | 0.1562 | 0.1605 |
| 28 | 0.0594 | 0.0846 | 0.1360 | 0.0872 | 0.1339 | 0.0890 | 0.1463 | 0.1538 | 0.0645 | 0.0959 | 0.1626 | 0.0960 | 0.1475 | 0.1012 | 0.1558 | 0.1602 |
| 29 | 0.0592 | 0.0842 | 0.1367 | 0.0870 | 0.1333 | 0.0888 | 0.1459 | 0.1533 | 0.0644 | 0.0955 | 0.1659 | 0.0958 | 0.1470 | 0.1011 | 0.1555 | 0.1599 |
| 30 | 0.0591 | 0.0838 | 0.1376 | 0.0868 | 0.1327 | 0.0886 | 0.1455 | 0.1528 | 0.0643 | 0.0953 | 0.1698 | 0.0956 | 0.1466 | 0.1009 | 0.1553 | 0.1596 |
| Mean | 0.0868 | 0.2859 | 0.2303 | 0.1389 | 0.3916 | 0.1302 | 0.2273 | 0.2275 | 0.0912 | 0.2674 | 0.2382 | 0.1466 | 0.3648 | 0.1405 | 0.2276 | 0.2277 |

Table 3. Natural mortality-at-age by sex for each estimator, based on the growth curve by Hsu et al. (2011).

| Hsu et al. (2011) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Male |  |  |  |  |  |  |  | Female |  |  |  |  |  |  |  |
| Age | Rev. Peterson \&Wroblewski (1984) | Gislason et al. <br> (2010) | $\begin{gathered} \hline \text { Chen \& } \\ \text { Watanabe } \\ (1989) \end{gathered}$ | Lorenzen (1996) | Charnovet al. (2012) | Previous (Peterson \& Wroblewski 1984) | Walter et al. <br> (2016) tc=0 | $\begin{gathered} \hline \text { Fujijami et al. } \\ \text { (2016) male } \\ \text { (Walters:tc=0) } \end{gathered}$ | Rev. Peterson \&Wroblewski (1984) | Gislason et al. <br> (2010) | $\begin{gathered} \hline \text { Chen \& } \\ \text { Watanabe } \\ (1989) \end{gathered}$ | Lorenzen (1996) | Charnov et al. <br> (2012) | Previous (Peterson \& Wroblewski 1984) | Walter et al. <br> (2016) tc=0 | Fujinami et al. (2016) female (Walters:tc=0) |
| 0 | 0.2316 | 1.3070 | 0.7059 | 0.4186 | 1.7050 | 0.3591 | 0.7191 | 0.7877 | 0.2613 | 1.8384 | 0.9792 | 0.4810 | 2.3365 | 0.3662 | 0.7274 | 0.8436 |
| 1 | 0.1646 | 0.6448 | 0.4552 | 0.2824 | 0.8828 | 0.2451 | 0.5069 | 0.5321 | 0.1667 | 0.7525 | 0.5623 | 0.2866 | 1.0166 | 0.2454 | 0.4788 | 0.5252 |
| 2 | 0.1330 | 0.4151 | 0.3462 | 0.2210 | 0.5856 | 0.1945 | 0.4025 | 0.4139 | 0.1301 | 0.4595 | 0.4139 | 0.2155 | 0.6420 | 0.1954 | 0.3730 | 0.3977 |
| 3 | 0.1145 | 0.3045 | 0.2856 | 0.1860 | 0.4388 | 0.1658 | 0.3406 | 0.3458 | 0.1107 | 0.3326 | 0.3386 | 0.1788 | 0.4751 | 0.1679 | 0.3147 | 0.3292 |
| 4 | 0.1024 | 0.2415 | 0.2473 | 0.1635 | 0.3535 | 0.1473 | 0.2998 | 0.3017 | 0.0986 | 0.2645 | 0.2937 | 0.1566 | 0.3837 | 0.1505 | 0.2782 | 0.2866 |
| 5 | 0.0938 | 0.2015 | 0.2210 | 0.1479 | 0.2987 | 0.1344 | 0.2710 | 0.2710 | 0.0905 | 0.2230 | 0.2641 | 0.1419 | 0.3273 | 0.1387 | 0.2535 | 0.2579 |
| 6 | 0.0875 | 0.1743 | 0.2020 | 0.1364 | 0.2609 | 0.1250 | 0.2498 | 0.2486 | 0.0848 | 0.1956 | 0.2435 | 0.1315 | 0.2898 | 0.1302 | 0.2358 | 0.2373 |
| 7 | 0.0827 | 0.1549 | 0.1877 | 0.1277 | 0.2337 | 0.1178 | 0.2337 | 0.2315 | 0.0805 | 0.1766 | 0.2285 | 0.1240 | 0.2634 | 0.1238 | 0.2228 | 0.2221 |
| 8 | 0.0788 | 0.1404 | 0.1766 | 0.1210 | 0.2133 | 0.1122 | 0.2210 | 0.2182 | 0.0773 | 0.1628 | 0.2172 | 0.1183 | 0.2441 | 0.1190 | 0.2129 | 0.2104 |
| 9 | 0.0758 | 0.1293 | 0.1678 | 0.1156 | 0.1976 | 0.1078 | 0.2108 | 0.2075 | 0.0748 | 0.1524 | 0.2086 | 0.1139 | 0.2297 | 0.1152 | 0.2052 | 0.2013 |
| 10 | 0.0733 | 0.1206 | 0.1607 | 0.1112 | 0.1852 | 0.1042 | 0.2026 | 0.1989 | 0.0728 | 0.1445 | 0.2018 | 0.1104 | 0.2186 | 0.1122 | 0.1992 | 0.1940 |
| 11 | 0.0712 | 0.1137 | 0.1549 | 0.1076 | 0.1753 | 0.1012 | 0.1958 | 0.1919 | 0.0713 | 0.1384 | 0.1967 | 0.1077 | 0.2099 | 0.1098 | 0.1943 | 0.1882 |
| 12 | 0.0695 | 0.1081 | 0.1501 | 0.1046 | 0.1672 | 0.0988 | 0.1902 | 0.1860 | 0.0700 | 0.1335 | 0.1929 | 0.1055 | 0.2030 | 0.1078 | 0.1905 | 0.1834 |
| 13 | 0.0681 | 0.1035 | 0.1461 | 0.1021 | 0.1606 | 0.0967 | 0.1855 | 0.1810 | 0.0690 | 0.1297 | 0.1903 | 0.1037 | 0.1975 | 0.1062 | 0.1873 | 0.1795 |
| 14 | 0.0668 | 0.0997 | 0.1427 | 0.1000 | 0.1550 | 0.0950 | 0.1815 | 0.1768 | 0.0681 | 0.1265 | 0.1889 | 0.1023 | 0.1931 | 0.1049 | 0.1847 | 0.1762 |
| 15 | 0.0658 | 0.0965 | 0.1399 | 0.0982 | 0.1504 | 0.0935 | 0.1781 | 0.1733 | 0.0675 | 0.1240 | 0.1886 | 0.1011 | 0.1895 | 0.1038 | 0.1826 | 0.1735 |
| 16 | 0.0649 | 0.0938 | 0.1376 | 0.0967 | 0.1465 | 0.0923 | 0.1752 | 0.1702 | 0.0669 | 0.1219 | 0.1893 | 0.1001 | 0.1865 | 0.1029 | 0.1809 | 0.1712 |
| 17 | 0.0641 | 0.0915 | 0.1358 | 0.0954 | 0.1431 | 0.0912 | 0.1727 | 0.1676 | 0.0664 | 0.1202 | 0.1912 | 0.0993 | 0.1841 | 0.1021 | 0.1794 | 0.1693 |
| 18 | 0.0635 | 0.0895 | 0.1343 | 0.0942 | 0.1403 | 0.0903 | 0.1705 | 0.1653 | 0.0660 | 0.1188 | 0.1942 | 0.0986 | 0.1821 | 0.1015 | 0.1782 | 0.1676 |
| 19 | 0.0629 | 0.0879 | 0.1333 | 0.0932 | 0.1378 | 0.0895 | 0.1686 | 0.1634 | 0.0657 | 0.1177 | 0.1985 | 0.0981 | 0.1804 | 0.1010 | 0.1772 | 0.1663 |
| 20 | 0.0624 | 0.0864 | 0.1326 | 0.0924 | 0.1357 | 0.0889 | 0.1670 | 0.1617 | 0.0654 | 0.1167 | 0.2043 | 0.0976 | 0.1791 | 0.1005 | 0.1764 | 0.1651 |
| 21 | 0.0620 | 0.0852 | 0.1324 | 0.0916 | 0.1339 | 0.0883 | 0.1656 | 0.1602 | 0.0652 | 0.1159 | 0.2117 | 0.0972 | 0.1779 | 0.1002 | 0.1757 | 0.1641 |
| 22 | 0.0616 | 0.0841 | 0.1324 | 0.0910 | 0.1323 | 0.0878 | 0.1644 | 0.1589 | 0.0650 | 0.1152 | 0.2212 | 0.0969 | 0.1770 | 0.0999 | 0.1752 | 0.1632 |
| 23 | 0.0612 | 0.0832 | 0.1329 | 0.0904 | 0.1309 | 0.0873 | 0.1633 | 0.1577 | 0.0649 | 0.1147 | 0.2332 | 0.0966 | 0.1762 | 0.0996 | 0.1747 | 0.1625 |
| 24 | 0.0610 | 0.0823 | 0.1337 | 0.0899 | 0.1297 | 0.0869 | 0.1624 | 0.1567 | 0.0647 | 0.1142 | 0.2484 | 0.0964 | 0.1755 | 0.0994 | 0.1743 | 0.1619 |
| 25 | 0.0607 | 0.0816 | 0.1349 | 0.0895 | 0.1287 | 0.0866 | 0.1616 | 0.1559 | 0.0646 | 0.1138 | 0.2679 | 0.0962 | 0.1749 | 0.0992 | 0.1739 | 0.1613 |
| 26 | 0.0605 | 0.0810 | 0.1365 | 0.0891 | 0.1278 | 0.0863 | 0.1608 | 0.1551 | 0.0645 | 0.1135 | 0.2933 | 0.0960 | 0.1745 | 0.0990 | 0.1736 | 0.1609 |
| 27 | 0.0603 | 0.0805 | 0.1386 | 0.0888 | 0.1270 | 0.0860 | 0.1602 | 0.1544 | 0.0644 | 0.1132 | 0.3272 | 0.0959 | 0.1741 | 0.0989 | 0.1734 | 0.1605 |
| 28 | 0.0601 | 0.0800 | 0.1411 | 0.0885 | 0.1263 | 0.0858 | 0.1596 | 0.1538 | 0.0644 | 0.1130 | 0.3742 | 0.0958 | 0.1737 | 0.0988 | 0.1732 | 0.1602 |
| 29 | 0.0599 | 0.0796 | 0.1441 | 0.0882 | 0.1256 | 0.0856 | 0.1591 | 0.1533 | 0.0643 | 0.1128 | 0.4427 | 0.0957 | 0.1735 | 0.0987 | 0.1730 | 0.1599 |
| 30 | 0.0598 | 0.0792 | 0.1477 | 0.0880 | 0.1251 | 0.0854 | 0.1587 | 0.1528 | 0.0643 | 0.1126 | 0.5510 | 0.0956 | 0.1732 | 0.0986 | 0.1729 | 0.1596 |
| Mean | 0.0808 | 0.1813 | 0.1915 | 0.1262 | 0.2630 | 0.1167 | 0.2277 | 0.2275 | 0.0829 | 0.2287 | 0.2922 | 0.1302 | 0.3252 | 0.1257 | 0.2282 | 0.2277 |



Figure 1. Growth curve by sex by Nakano (1994), Hsu et al. (2011), and Fujinami et al. (2016).

[^0]


Figure 2. Natural mortality at age for male (upper) and female (below) blue shark by each estimator, based on the growth curve by Nakano (1994).

[^1]

Figure 3. Natural mortality at age for male (upper) and female (below) blue shark by each estimator, based on the growth curve by Hsu et al. (2011).

[^2]

Figure 4. Natural mortality at age for male (upper) and female (below) blue shark by each $t_{c}$, based on the growth curve by Nakano (1994).

[^3]


Figure 5. Natural mortality at age for male (upper) and female (below) blue shark by each $t_{c}$, based on the growth curve by Nakano (1994) with assumption of $t_{\max }=20$.

[^4]Appendix 1. Concept of Method 2 in Walter et al. (2016) and derivation of equation (10) in Methods.

## Concept of Method 2 by Walter et al. (2016)

This method consists of 2 step;

Step1: M per unit length (M_unitL: theoretical value) is calculated based on Target $M$.
Step2: Age-specific $M$ is estimated based on the length at age using VBGC.

M_unitL depends on $t_{c}$ and $t_{\max }$. Only VBGC is available for growth curve at present.

As geometric interpretation of this method, one can distribute the area of rectangle encompassed by Target $\mathrm{M}, t_{c}$ and $t_{\text {max }}$ for each age (in originally meaning, length). Thus, the area of red color is equal to that of green color in the figure below. Thereby, survival rate at $t_{\max }$ is guarantee to be equal to that when Target M is applied.


Appendix Figure 1. Diagram of Method 2 in Walter et al. (2016). Solid curve is age specific M with $t_{c}=0$.

## Derivation of equation (10)

According to the equation for estimating $M$ which takes into account the variation by growth (Lorenzen 2000, 2005), Survival rate $\left(\mathrm{S}_{t}\right)$ between $t_{c}$ and $t_{\max }$ can be expressed as follows,

$$
\begin{equation*}
S=\left(\frac{l_{c}}{l_{c}+L_{\infty}\left(\exp \left(K\left(t_{\max }-t_{c}\right)\right)-1\right)}\right)^{\frac{M_{r} l_{r}}{L_{\infty} k}} \tag{1}
\end{equation*}
$$

, where $L_{\infty}$ is asymptotic length, $K$ is von Bertalanffy growth coefficient, $M_{r}$ is M by unit length $\left(l_{r}\right) . l_{c}$ is the

[^5]body length at $t_{c}$. When VBGC is substituted into $l_{c}$ and then equation (1) can be transformed as follows,
\[

$$
\begin{equation*}
l_{c}=L_{\infty}\left(1-\exp \left(-\mathrm{K}\left(t_{c}-t_{o}\right)\right)\right) \tag{2}
\end{equation*}
$$

\]

Given that mean of natural mortality between $t_{c}$ and $t_{\max }$ is expressed as Target $\mathrm{M}\left(M_{T}\right)$, survival rate ( $S$ ) between $t_{c}$ and $t_{\text {max }}$ is expressed as follows,

$$
\begin{equation*}
S=\exp \left(-M_{T}\left(t_{\max }-t_{c}\right)\right) \tag{3}
\end{equation*}
$$

Assuming that survival rate $(S)$ derived from $M$ (equation 1 ) and that $(S)$ derived from $M_{T}$ (equation 3) is equal,

$$
\begin{equation*}
S=\exp \left(-M_{T}\left(t_{\max }-t_{c}\right)\right)=\left(\frac{l_{c}}{l_{c}+L_{\infty}\left(\exp \left(K\left(t_{\max }-t_{c}\right)\right)-1\right)}\right)^{\frac{M_{r} l_{r}}{L_{\infty} k}} \tag{4}
\end{equation*}
$$

,which can be transformed as follows,

$$
\begin{equation*}
M_{r} l_{r}=\frac{-M_{T} L_{\infty} K\left(t_{\max }-t_{c}\right)}{\ln \left(\frac{l_{c}}{l_{c}+L_{\infty}\left(\exp \left(K\left(t_{\max }-t_{c}\right)\right)-1\right)}\right)} \tag{5}
\end{equation*}
$$

Same as derivation of equation 1 , survival rate $\left(S_{t}\right)$ between $t$ and $t+1$ is expressed as

$$
\begin{equation*}
\mathrm{S}_{t}=\left(\frac{l_{t}}{l_{t}+L_{\infty}(\exp (K)-1)}\right)^{\frac{M_{r} l_{r}}{L_{\infty} k}} \tag{6}
\end{equation*}
$$

Because the relationship between survival rate and natural mortality can be described as

$$
\begin{equation*}
\mathrm{S}_{t}=\exp \left(-M_{t}\right) \tag{7}
\end{equation*}
$$

, where $M_{t}$ denotes natural mortality at age $t$. From equation (6) and (7), natural mortality is expressed as

$$
\begin{equation*}
M_{t}=\frac{-M_{r} l_{r}}{L_{\infty} K} \ln \left(\frac{l_{t}}{l_{t}+L_{\infty}(\exp (K)-1)}\right) \tag{8}
\end{equation*}
$$

If equation (5) is substituted in equation (8),

$$
\begin{equation*}
M_{t}=\frac{M_{T}\left(t_{\max }-t_{c}\right)}{\ln \left(\frac{l_{c}}{l_{c}+L_{\infty}\left(\exp \left(K\left(t_{\max }-t_{c}\right)\right)-1\right)}\right)} \ln \left(\frac{l_{t}}{l_{t}+L_{\infty}(\exp (K)-1)}\right) \tag{9}
\end{equation*}
$$

is derived.

[^6]Appendix 2. Natural mortality-at-age by sex estimated by the method of Walter et al. (2016) for varioust ${ }_{c}$, based on the growth curve by Nakano (1994).

|  |  | Nakano (male) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Age | tc $=0$ | tc $=1$ | tc $=2$ | tc $=3$ | tc $=4$ | tc $=5$ |  |
| 0 | 1.0072 | 1.1401 | 1.2197 | 1.2768 | 1.3211 | 1.3569 |  |
| 1 | 0.5744 | 0.6502 | 0.6955 | 0.7281 | 0.7534 | 0.7738 |  |
| 2 | 0.4195 | 0.4748 | 0.5080 | 0.5318 | 0.5502 | 0.5651 |  |
| 3 | 0.3395 | 0.3843 | 0.4112 | 0.4304 | 0.4453 | 0.4574 |  |
| 4 | 0.2910 | 0.3293 | 0.3523 | 0.3688 | 0.3816 | 0.3919 |  |
| 5 | 0.2585 | 0.2926 | 0.3130 | 0.3277 | 0.3390 | 0.3482 |  |
| 6 | 0.2354 | 0.2665 | 0.2851 | 0.2984 | 0.3088 | 0.3171 |  |
| 7 | 0.2183 | 0.2471 | 0.2643 | 0.2767 | 0.2863 | 0.2941 |  |
| 8 | 0.2052 | 0.2323 | 0.2485 | 0.2601 | 0.2691 | 0.2764 |  |
| 9 | 0.1949 | 0.2206 | 0.2360 | 0.2471 | 0.2557 | 0.2626 |  |
| 10 | 0.1867 | 0.2113 | 0.2261 | 0.2367 | 0.2449 | 0.2515 |  |
| 11 | 0.1800 | 0.2038 | 0.2180 | 0.2282 | 0.2361 | 0.2425 |  |
| 12 | 0.1745 | 0.1976 | 0.2114 | 0.2213 | 0.2289 | 0.2351 |  |
| 13 | 0.1700 | 0.1924 | 0.2058 | 0.2155 | 0.2230 | 0.2290 |  |
| 14 | 0.1662 | 0.1881 | 0.2012 | 0.2107 | 0.2180 | 0.2239 |  |
| 15 | 0.1630 | 0.1845 | 0.1974 | 0.2066 | 0.2138 | 0.2196 |  |
| 16 | 0.1603 | 0.1814 | 0.1941 | 0.2032 | 0.2102 | 0.2159 |  |
| 17 | 0.1579 | 0.1788 | 0.1913 | 0.2002 | 0.2072 | 0.2128 |  |
| 18 | 0.1560 | 0.1765 | 0.1889 | 0.1977 | 0.2046 | 0.2101 |  |
| 19 | 0.1543 | 0.1746 | 0.1868 | 0.1955 | 0.2023 | 0.2078 |  |
| 20 | 0.1528 | 0.1730 | 0.1850 | 0.1937 | 0.2004 | 0.2058 |  |
| 21 | 0.1515 | 0.1715 | 0.1835 | 0.1921 | 0.1988 | 0.2041 |  |
| 22 | 0.1504 | 0.1703 | 0.1822 | 0.1907 | 0.1973 | 0.2027 |  |
| 23 | 0.1495 | 0.1692 | 0.1810 | 0.1895 | 0.1961 | 0.2014 |  |
| 24 | 0.1487 | 0.1683 | 0.1800 | 0.1885 | 0.1950 | 0.2003 |  |
| 25 | 0.1479 | 0.1675 | 0.1792 | 0.1875 | 0.1941 | 0.1993 |  |
| 26 | 0.1473 | 0.1668 | 0.1784 | 0.1868 | 0.1932 | 0.1985 |  |
| 27 | 0.1468 | 0.1661 | 0.1777 | 0.1861 | 0.1925 | 0.1977 |  |
| 28 | 0.1463 | 0.1656 | 0.1772 | 0.1855 | 0.1919 | 0.1971 |  |
| 29 | 0.1459 | 0.1651 | 0.1767 | 0.1849 | 0.1913 | 0.1965 |  |
| 30 | 0.1455 | 0.1647 | 0.1762 | 0.1845 | 0.1909 | 0.1960 |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |


|  | Nakano (Female) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Age | $\mathrm{tc}=0$ | $\mathrm{tc}=1$ | $\mathrm{tc}=2$ | $\mathrm{tc}=3$ | $\mathrm{tc}=4$ | $\mathrm{tc}=5$ |
| 0 | 0.9093 | 1.0124 | 1.0748 | 1.1193 | 1.1536 | 1.1810 |
| 1 | 0.5421 | 0.6036 | 0.6408 | 0.6674 | 0.6878 | 0.7041 |
| 2 | 0.4038 | 0.4495 | 0.4772 | 0.4970 | 0.5122 | 0.5244 |
| 3 | 0.3310 | 0.3686 | 0.3913 | 0.4075 | 0.4200 | 0.4299 |
| 4 | 0.2865 | 0.3190 | 0.3386 | 0.3527 | 0.3634 | 0.3721 |
| 5 | 0.2566 | 0.2857 | 0.3033 | 0.3159 | 0.3256 | 0.3333 |
| 6 | 0.2354 | 0.2621 | 0.2782 | 0.2898 | 0.2986 | 0.3057 |
| 7 | 0.2197 | 0.2446 | 0.2597 | 0.2704 | 0.2787 | 0.2853 |
| 8 | 0.2077 | 0.2312 | 0.2455 | 0.2557 | 0.2635 | 0.2697 |
| 9 | 0.1983 | 0.2208 | 0.2344 | 0.2441 | 0.2516 | 0.2576 |
| 10 | 0.1908 | 0.2125 | 0.2256 | 0.2349 | 0.2421 | 0.2479 |
| 11 | 0.1848 | 0.2058 | 0.2185 | 0.2275 | 0.2345 | 0.2400 |
| 12 | 0.1799 | 0.2003 | 0.2126 | 0.2215 | 0.2282 | 0.2337 |
| 13 | 0.1759 | 0.1958 | 0.2079 | 0.2165 | 0.2231 | 0.2284 |
| 14 | 0.1725 | 0.1921 | 0.2039 | 0.2123 | 0.2188 | 0.2240 |
| 15 | 0.1697 | 0.1889 | 0.2006 | 0.2089 | 0.2153 | 0.2204 |
| 16 | 0.1673 | 0.1863 | 0.1978 | 0.2060 | 0.2123 | 0.2173 |
| 17 | 0.1653 | 0.1841 | 0.1954 | 0.2035 | 0.2098 | 0.2147 |
| 18 | 0.1637 | 0.1822 | 0.1934 | 0.2015 | 0.2076 | 0.2126 |
| 19 | 0.1622 | 0.1806 | 0.1918 | 0.1997 | 0.2058 | 0.2107 |
| 20 | 0.1610 | 0.1793 | 0.1903 | 0.1982 | 0.2043 | 0.2091 |
| 21 | 0.1600 | 0.1781 | 0.1891 | 0.1969 | 0.2029 | 0.2078 |
| 22 | 0.1591 | 0.1771 | 0.1880 | 0.1958 | 0.2018 | 0.2066 |
| 23 | 0.1583 | 0.1763 | 0.1871 | 0.1949 | 0.2008 | 0.2056 |
| 24 | 0.1577 | 0.1755 | 0.1864 | 0.1941 | 0.2000 | 0.2048 |
| 25 | 0.1571 | 0.1749 | 0.1857 | 0.1934 | 0.1993 | 0.2040 |
| 26 | 0.1566 | 0.1744 | 0.1851 | 0.1928 | 0.1987 | 0.2034 |
| 27 | 0.1562 | 0.1739 | 0.1846 | 0.1923 | 0.1982 | 0.2029 |
| 28 | 0.1558 | 0.1735 | 0.1842 | 0.1918 | 0.1977 | 0.2024 |
| 29 | 0.1555 | 0.1732 | 0.1838 | 0.1915 | 0.1973 | 0.2020 |
| 30 | 0.1553 | 0.1729 | 0.1835 | 0.1911 | 0.1970 | 0.2017 |
|  |  |  |  |  |  |  |
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Appendix 3. Natural mortality-at-age by sex estimated by the method of Walter et al. (2016) for various $t_{\text {max }}$, based on three VBGC with assumption of $t_{c}=0$.

|  |  |  |  |
| :---: | :---: | :---: | :---: |
|  | tmax $=30$ |  |  |
| Age | Fujinami et al. <br> 2016 | Nakano (1994) | Hsu et al. <br> $(2011)$ |
| 0 | 0.8436 | 0.9093 | 0.7274 |
| 1 | 0.5252 | 0.5421 | 0.4788 |
| 2 | 0.3977 | 0.4038 | 0.3730 |
| 3 | 0.3292 | 0.3310 | 0.3147 |
| 4 | 0.2866 | 0.2865 | 0.2782 |
| 5 | 0.2579 | 0.2566 | 0.2535 |
| 6 | 0.2373 | 0.2354 | 0.2358 |
| 7 | 0.2221 | 0.2197 | 0.2228 |
| 8 | 0.2104 | 0.2077 | 0.2129 |
| 9 | 0.2013 | 0.1983 | 0.2052 |
| 10 | 0.1940 | 0.1908 | 0.1992 |
| 11 | 0.1882 | 0.1848 | 0.1943 |
| 12 | 0.1834 | 0.1799 | 0.1905 |
| 13 | 0.1795 | 0.1759 | 0.1873 |
| 14 | 0.1762 | 0.1725 | 0.1847 |
| 15 | 0.1735 | 0.1697 | 0.1826 |
| 16 | 0.1712 | 0.1673 | 0.1809 |
| 17 | 0.1693 | 0.1653 | 0.1794 |
| 18 | 0.1676 | 0.1637 | 0.1782 |
| 19 | 0.1663 | 0.1622 | 0.1772 |
| 20 | 0.1651 | 0.1610 | 0.1764 |
| 21 | 0.1641 | 0.1600 | 0.1757 |
| 22 | 0.1632 | 0.1591 | 0.1752 |
| 23 | 0.1625 | 0.1583 | 0.1747 |
| 24 | 0.1619 | 0.1577 | 0.1743 |
| 25 | 0.1613 | 0.1571 | 0.1739 |
| 26 | 0.1609 | 0.1566 | 0.1736 |
| 27 | 0.1605 | 0.1562 | 0.1734 |
| 28 | 0.1602 | 0.1558 | 0.1732 |
| 29 | 0.1599 | 0.1555 | 0.1730 |
| 30 | 0.1596 | 0.1553 | 0.1729 |
|  |  |  |  |


|  |  |  |  |
| :---: | :---: | :---: | :---: |
| tmax $=24$ |  |  |  |
| Age | Fujinami et al. <br> 2016 | Nakano (1994) | Hsu et al. <br> $(2011)$ |
| 0 | 0.7846 | 0.8420 | 0.6854 |
| 1 | 0.4884 | 0.5020 | 0.4511 |
| 2 | 0.3699 | 0.3739 | 0.3514 |
| 3 | 0.3061 | 0.3065 | 0.2965 |
| 4 | 0.2666 | 0.2653 | 0.2621 |
| 5 | 0.2398 | 0.2376 | 0.2388 |
| 6 | 0.2207 | 0.2180 | 0.2222 |
| 7 | 0.2066 | 0.2034 | 0.2099 |
| 8 | 0.1957 | 0.1923 | 0.2006 |
| 9 | 0.1872 | 0.1836 | 0.1933 |
| 10 | 0.1805 | 0.1767 | 0.1876 |
| 11 | 0.1750 | 0.1711 | 0.1831 |
| 12 | 0.1706 | 0.1666 | 0.1794 |
| 13 | 0.1669 | 0.1629 | 0.1765 |
| 14 | 0.1639 | 0.1597 | 0.1741 |
| 15 | 0.1613 | 0.1571 | 0.1721 |
| 16 | 0.1592 | 0.1550 | 0.1704 |
| 17 | 0.1574 | 0.1531 | 0.1691 |
| 18 | 0.1559 | 0.1516 | 0.1679 |
| 19 | 0.1546 | 0.1502 | 0.1670 |
| 20 | 0.1535 | 0.1491 | 0.1662 |
| 21 | 0.1526 | 0.1481 | 0.1656 |
| 22 | 0.1518 | 0.1473 | 0.1650 |
| 23 | 0.1511 | 0.1466 | 0.1646 |
| 24 | 0.1506 | 0.1460 | 0.1642 |
|  |  |  |  |
|  |  |  |  |


|  |  |  |  |
| :---: | :---: | :---: | :---: |
| tmax $=20$ |  |  |  |
| Age | Fujinami et al. <br> 2016 | Nakano (1994) | Hsu et al. <br> $(2011)$ |
| 0 | 0.7349 | 0.7858 | 0.6489 |
| 1 | 0.4575 | 0.4685 | 0.4271 |
| 2 | 0.3465 | 0.3489 | 0.3327 |
| 3 | 0.2867 | 0.2861 | 0.2807 |
| 4 | 0.2497 | 0.2476 | 0.2482 |
| 5 | 0.2246 | 0.2218 | 0.2261 |
| 6 | 0.2068 | 0.2034 | 0.2104 |
| 7 | 0.1935 | 0.1899 | 0.1987 |
| 8 | 0.1833 | 0.1795 | 0.1899 |
| 9 | 0.1754 | 0.1714 | 0.1831 |
| 10 | 0.1690 | 0.1649 | 0.1777 |
| 11 | 0.1639 | 0.1597 | 0.1734 |
| 12 | 0.1598 | 0.1555 | 0.1699 |
| 13 | 0.1563 | 0.1520 | 0.1671 |
| 14 | 0.1535 | 0.1491 | 0.1648 |
| 15 | 0.1511 | 0.1467 | 0.1629 |
| 16 | 0.1491 | 0.1446 | 0.1614 |
| 17 | 0.1475 | 0.1429 | 0.1601 |
| 18 | 0.1460 | 0.1414 | 0.1590 |
| 19 | 0.1448 | 0.1402 | 0.1581 |
| 20 | 0.1438 | 0.1392 | 0.1574 |
|  |  |  |  |

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Appendix 4. Natural mortality-at-age by sex estimated by the method of Walter et al. (2016) for various $t_{c}$, based on the growth curve by Fujinami et al. (2016).

| Fujinami (male) |  |  |  |  |  |  | Fujinami (female) |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Age | tc=0 | tc=1 | tc=2 | tc=3 | tc=4 | tc=5 | Age | tc $=0$ | tc=1 | tc=2 | tc=3 | tc=4 | tc=5 |
| 0 | 0.7877 | 0.8595 | 0.9090 | 0.9468 | 0.9771 | 1.0023 | 0 | 0.8436 | 0.9291 | 0.9822 | 1.0205 | 1.0501 | 1.0737 |
| 1 | 0.5321 | 0.5807 | 0.6141 | 0.6396 | 0.6601 | 0.6771 | 1 | 0.5252 | 0.5784 | 0.6115 | 0.6353 | 0.6537 | 0.6684 |
| 2 | 0.4139 | 0.4516 | 0.4777 | 0.4975 | 0.5134 | 0.5266 | 2 | 0.3977 | 0.4380 | 0.4631 | 0.4811 | 0.4951 | 0.5062 |
| 3 | 0.3458 | 0.3773 | 0.3991 | 0.4157 | 0.4290 | 0.4400 | 3 | 0.3292 | 0.3625 | 0.3832 | 0.3982 | 0.4097 | 0.4189 |
| 4 | 0.3017 | 0.3293 | 0.3482 | 0.3627 | 0.3743 | 0.3839 | 4 | 0.2866 | 0.3156 | 0.3337 | 0.3467 | 0.3567 | 0.3648 |
| 5 | 0.2710 | 0.2958 | 0.3128 | 0.3258 | 0.3362 | 0.3449 | 5 | 0.2579 | 0.2840 | 0.3002 | 0.3120 | 0.3210 | 0.3282 |
| 6 | 0.2486 | 0.2712 | 0.2869 | 0.2988 | 0.3083 | 0.3163 | 6 | 0.2373 | 0.2614 | 0.2763 | 0.2871 | 0.2954 | 0.3021 |
| 7 | 0.2315 | 0.2526 | 0.2672 | 0.2783 | 0.2872 | 0.2946 | 7 | 0.2221 | 0.2446 | 0.2586 | 0.2687 | 0.2764 | 0.2827 |
| 8 | 0.2182 | 0.2381 | 0.2518 | 0.2622 | 0.2706 | 0.2776 | 8 | 0.2104 | 0.2318 | 0.2450 | 0.2546 | 0.2619 | 0.2678 |
| 9 | 0.2075 | 0.2265 | 0.2395 | 0.2495 | 0.2575 | 0.2641 | 9 | 0.2013 | 0.2217 | 0.2344 | 0.2435 | 0.2506 | 0.2562 |
| 10 | 0.1989 | 0.2171 | 0.2296 | 0.2391 | 0.2468 | 0.2531 | 10 | 0.1940 | 0.2137 | 0.2259 | 0.2347 | 0.2415 | 0.2470 |
| 11 | 0.1919 | 0.2094 | 0.2214 | 0.2306 | 0.2380 | 0.2441 | 11 | 0.1882 | 0.2072 | 0.2191 | 0.2276 | 0.2342 | 0.2395 |
| 12 | 0.1860 | 0.2029 | 0.2146 | 0.2235 | 0.2307 | 0.2366 | 12 | 0.1834 | 0.2020 | 0.2135 | 0.2219 | 0.2283 | 0.2334 |
| 13 | 0.1810 | 0.1975 | 0.2089 | 0.2176 | 0.2246 | 0.2303 | 13 | 0.1795 | 0.1976 | 0.2089 | 0.2171 | 0.2234 | 0.2284 |
| 14 | 0.1768 | 0.1930 | 0.2041 | 0.2126 | 0.2194 | 0.2250 | 14 | 0.1762 | 0.1940 | 0.2051 | 0.2131 | 0.2193 | 0.2243 |
| 15 | 0.1733 | 0.1891 | 0.2000 | 0.2083 | 0.2150 | 0.2205 | 15 | 0.1735 | 0.1911 | 0.2020 | 0.2099 | 0.2159 | 0.2208 |
| 16 | 0.1702 | 0.1858 | 0.1965 | 0.2046 | 0.2112 | 0.2166 | 16 | 0.1712 | 0.1885 | 0.1993 | 0.2071 | 0.2131 | 0.2179 |
| 17 | 0.1676 | 0.1829 | 0.1934 | 0.2015 | 0.2079 | 0.2133 | 17 | 0.1693 | 0.1864 | 0.1971 | 0.2048 | 0.2107 | 0.2154 |
| 18 | 0.1653 | 0.1804 | 0.1908 | 0.1987 | 0.2051 | 0.2104 | 18 | 0.1676 | 0.1846 | 0.1952 | 0.2028 | 0.2087 | 0.2134 |
| 19 | 0.1634 | 0.1783 | 0.1885 | 0.1964 | 0.2027 | 0.2079 | 19 | 0.1663 | 0.1831 | 0.1936 | 0.2011 | 0.2069 | 0.2116 |
| 20 | 0.1617 | 0.1764 | 0.1866 | 0.1943 | 0.2006 | 0.2057 | 20 | 0.1651 | 0.1818 | 0.1922 | 0.1997 | 0.2055 | 0.2101 |
| 21 | 0.1602 | 0.1748 | 0.1849 | 0.1925 | 0.1987 | 0.2038 | 21 | 0.1641 | 0.1807 | 0.1910 | 0.1985 | 0.2042 | 0.2088 |
| 22 | 0.1589 | 0.1734 | 0.1834 | 0.1910 | 0.1971 | 0.2022 | 22 | 0.1632 | 0.1798 | 0.1901 | 0.1975 | 0.2032 | 0.2078 |
| 23 | 0.1577 | 0.1721 | 0.1820 | 0.1896 | 0.1957 | 0.2007 | 23 | 0.1625 | 0.1790 | 0.1892 | 0.1966 | 0.2023 | 0.2068 |
| 24 | 0.1567 | 0.1710 | 0.1809 | 0.1884 | 0.1944 | 0.1994 | 24 | 0.1619 | 0.1783 | 0.1885 | 0.1958 | 0.2015 | 0.2060 |
| 25 | 0.1559 | 0.1701 | 0.1799 | 0.1873 | 0.1933 | 0.1983 | 25 | 0.1613 | 0.1777 | 0.1879 | 0.1952 | 0.2008 | 0.2054 |
| 26 | 0.1551 | 0.1692 | 0.1790 | 0.1864 | 0.1924 | 0.1973 | 26 | 0.1609 | 0.1772 | 0.1873 | 0.1946 | 0.2003 | 0.2048 |
| 27 | 0.1544 | 0.1685 | 0.1782 | 0.1856 | 0.1915 | 0.1965 | 27 | 0.1605 | 0.1768 | 0.1869 | 0.1942 | 0.1998 | 0.2043 |
| 28 | 0.1538 | 0.1678 | 0.1775 | 0.1849 | 0.1908 | 0.1957 | 28 | 0.1602 | 0.1764 | 0.1865 | 0.1937 | 0.1993 | 0.2038 |
| 29 | 0.1533 | 0.1672 | 0.1769 | 0.1842 | 0.1901 | 0.1950 | 29 | 0.1599 | 0.1761 | 0.1861 | 0.1934 | 0.1990 | 0.2035 |
| 30 | 0.1528 | 0.1667 | 0.1763 | 0.1837 | 0.1895 | 0.1944 | 30 | 0.1596 | 0.1758 | 0.1858 | 0.1931 | 0.1987 | 0.2032 |

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