A Length-Based Catch Curve for Mulitgear Fisheries

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Motivation

- Use length frequency data (as LBSPR with variable growth)
- Bayesian fitting
- Allow flexible selectivity functions and multiple gears

Assumptions

1. **The population has been in an approximate steady-state for a generation or more around when the length sampling takes place (No time series).**
2. Mean growth follows the von Bertalanffy growth curve.
3. Fish asymptotic size (L_∞) is Gamma distributed for individual fish which governs growth variability.
4. Mortality fixed within each length interval. It can vary arbitrarily between intervals.
5. **Length data are representative of the catch length composition and the relative total catch numbers for each selectivity group.**

MODEL DESCRIPTION

Interval Mortality

Given its asymptotic length and a fixed mortality rate ($Z/k = Z_k$), the probability for survival for an individual fish passing through a length bin is:

$$e^{-Z_k\, t_{ki}} = NA \qquad L_\infty \le L_i$$

$$e^{-Z_k\, t_{ki}} = 0 \qquad L_i < L_\infty \le L_{i+1}$$

$$e^{-Z_k\, t_{ki}} = \left(\frac{L_\infty - L_i}{L_\infty - L_{i+1}}\right)^{-Z_k} \qquad L_{i+1} < L_\infty$$

Survival Model

If the mortality can be defined for sequential length intervals each with a fixed mortality, the survival for particular fish starting at length L_0 to length L_n can be defined as the product of surviving each interval between:

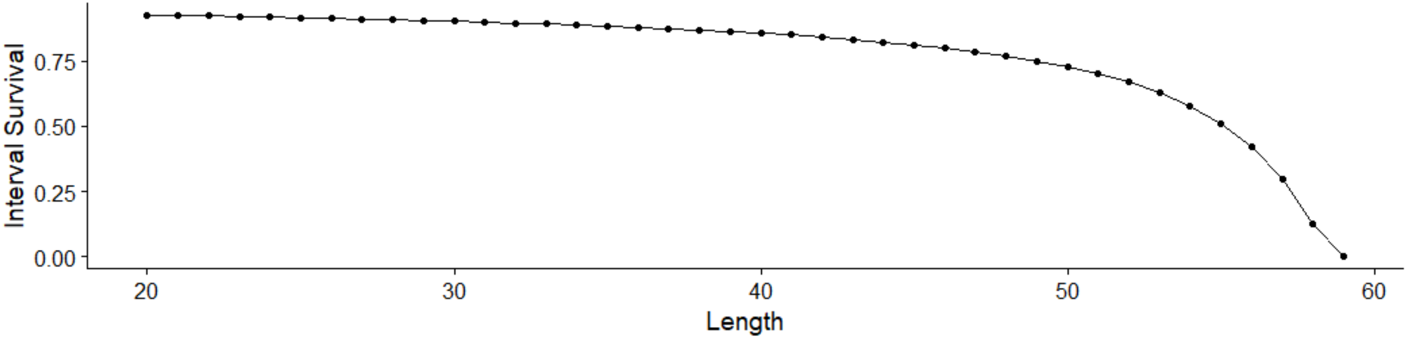
$$S_n = \left(\frac{L_\infty - L_0}{L_\infty - L_1}\right)^{-Z_1} \left(\frac{L_\infty - L_1}{L_\infty - L_2}\right)^{-Z_2} \cdots \left(\frac{L_\infty - L_{n-2}}{L_\infty - L_{n-1}}\right)^{-Z_{n-1}} \left(\frac{L_\infty - L_{n-1}}{L_\infty - L_n}\right)^{-Z_n}$$

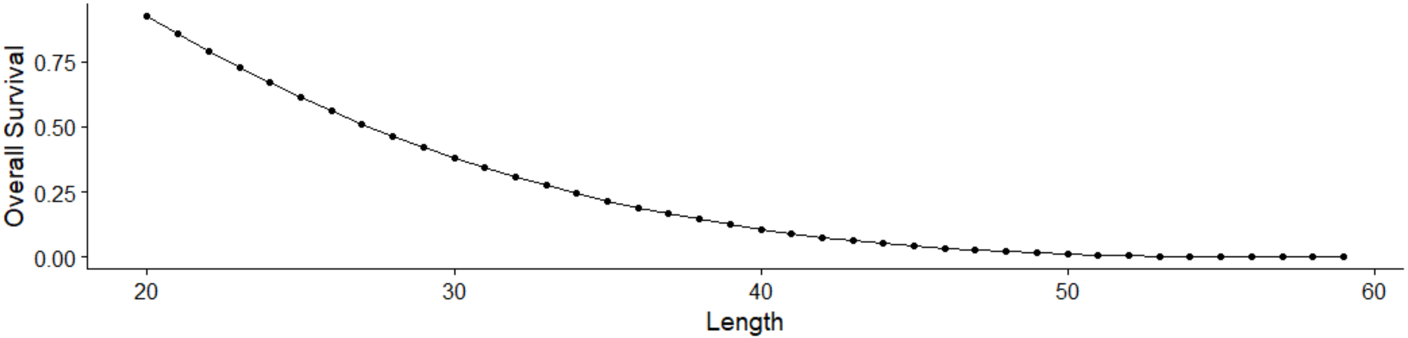
$$= (L_\infty - L_0)^{-Z_1} (L_\infty - L_1)^{Z_1 - Z_2} (L_\infty - L_2)^{Z_2 - Z_3} \ldots (L_\infty - L_{n-1})^{Z_{n-1} - Z_n} (L_\infty - L_n)^{Z_n}$$

$$= (L_\infty - L_0)^{-Z_1} \; (L_\infty - L_n)^{Z_n} \prod_{i=1}^{n-1} (L_\infty - L_i)^{Z_i - Z_{i+1}}$$

Survival Model

Fixed Z/k





Gamma Growth Variation

$$\Pr(\mathrm{L}_\infty) \; = \frac{\beta^{\alpha}}{\Gamma(\alpha)} {L_\infty}^{\alpha-1} \; e^{-\beta \, L_\infty}$$

Combined Model

The probability that fish will survive to length interval n with lower bound L_n is given by:

$$S_n = \int_{L_n}^{\infty} \frac{\beta^{\alpha}}{\Gamma(\alpha)} L_{\infty}{}^{\alpha-1}\, e^{-\beta\, L_{\infty}} (L_{\infty} - L_0)^{-Z_1}\, (L_{\infty} - L_n)^{Z_n} \prod_{i=1}^{n-1} (L_{\infty} - L_i)^{Z_i - Z_{i+1}}\, dL_{\infty}$$

Gauss-Laguerre quadrature is used for numerical integration (fast and accurate in this case).

Expected Population and Catch-at-length

Assuming constant recruitment, the numbers of fish within each interval can be estimated by integrating over the time interval:

$$N_n = \frac{S_n - S_{n+1}}{Z_n}$$

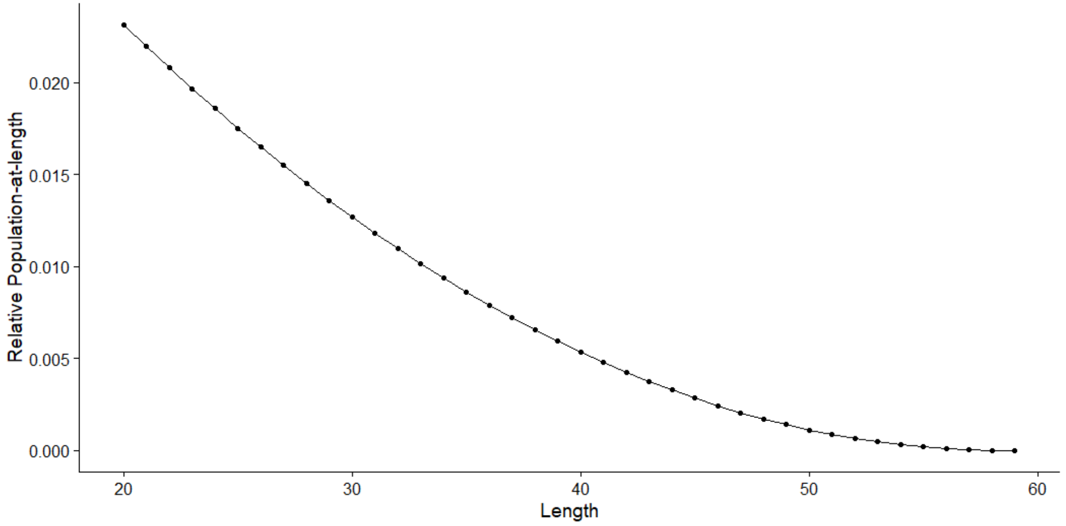
$$C_n = \frac{F_n}{Z_n}(S_n - S_{n+1})$$

where C_n is catch as a proportion of mortality and is proportional to the expected number of fish in a length frequency sample in length interval n. This is the standard catch equation used in VPA, for example.

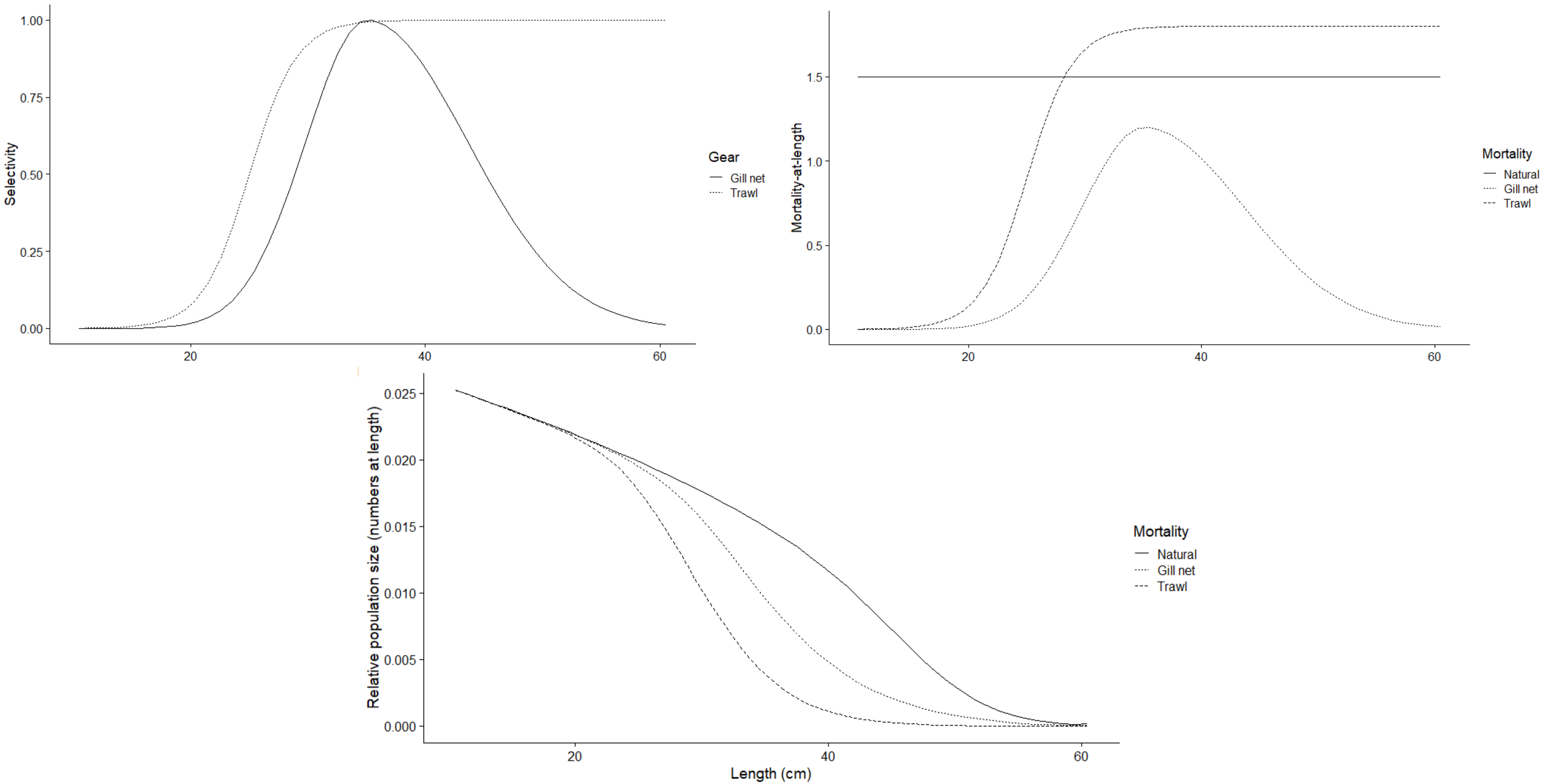
F_n will reflect the selectivity pattern.

Several gears can be represented each with its own F_n

Length intervals do not have to be regular.



Multiple Gears



APPLICATION

Application R package: *fishblicc*

- Work-In-Progress
- Implements Bayesian length-based catch curve (in Stan)
- Selectivity models based on **simple mixtures** of logistic, normal, single-sided normal and double-sided normal

- R Package: https://github.com/**PaulAHMedley/fishblicc**

Priors

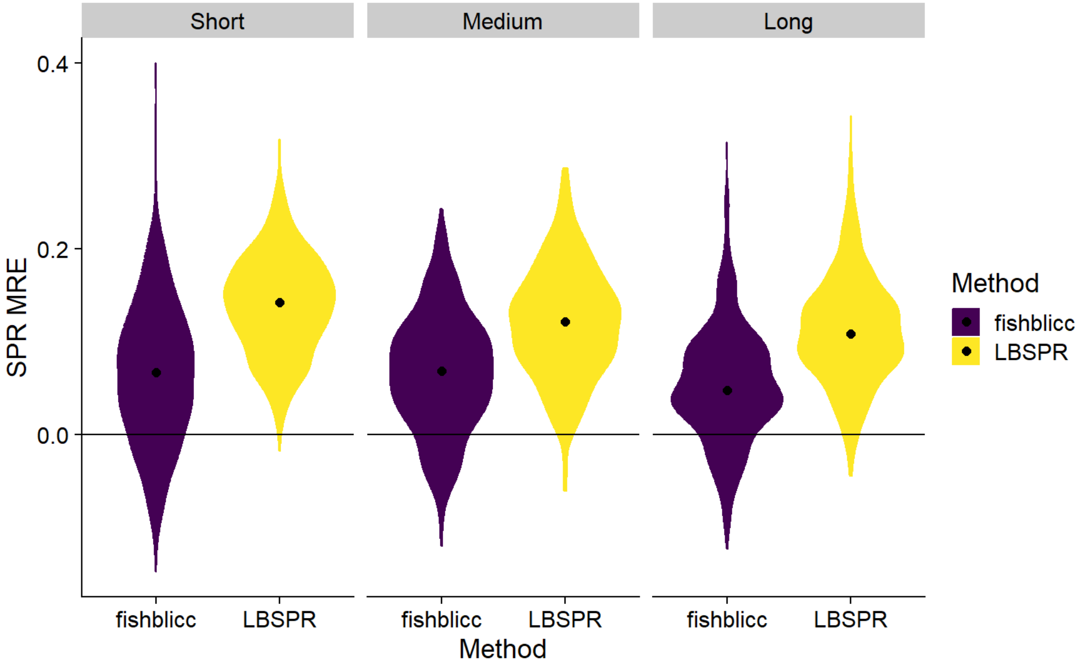
- Informative priors:
  - L_∞ is required - often available from Fishbase or elsewhere
  - Natural mortality: M ≈ 1.5 K
  - Growth CV (Galpha) - default 10%
  - K and t_0 not required
- Non-informative priors (usually)
  - Fishing mortality / selectivity parameters
  - Observation error

SIMULATION TESTING

fishblicc vs LBSPR

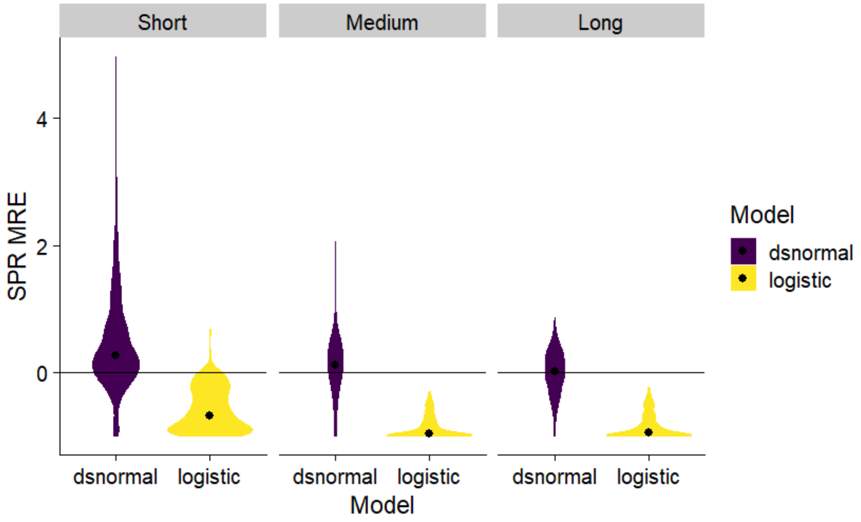
- Chong, Lisa, Tobias K Mildenberger, Merrill B Rudd, Marc H Taylor, Jason M Cope, Trevor A Branch, Matthias Wolff, and Moritz Stäbler. 2019. "Performance Evaluation of Data-Limited, Length-Based Stock Assessment Methods." Edited by Emory Anderson. *ICES Journal of Marine Science* 77 (1): 97–108. https://doi.org/10.1093/icesjms/fsz212
- Individual Based Model for a fish population originally used to test ELEFAN
- 3 representative species
- Mean relative error for SPR for 500 simulations with monthly samples taken for the three life histories.

Logistic Selectivity

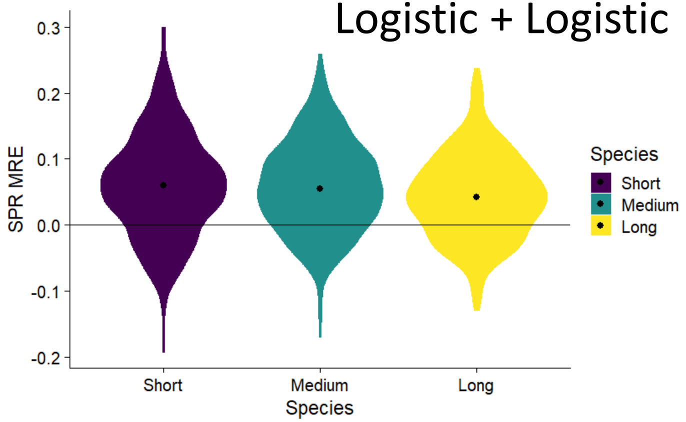


Multiple Selectivities

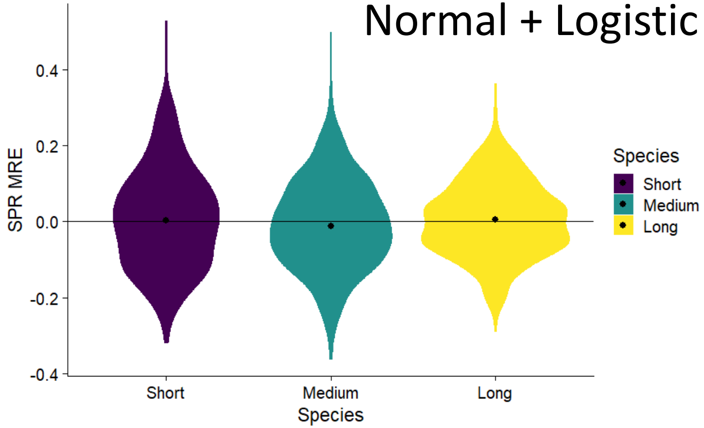
Double-sided normal selectivity



Logistic + Logistic

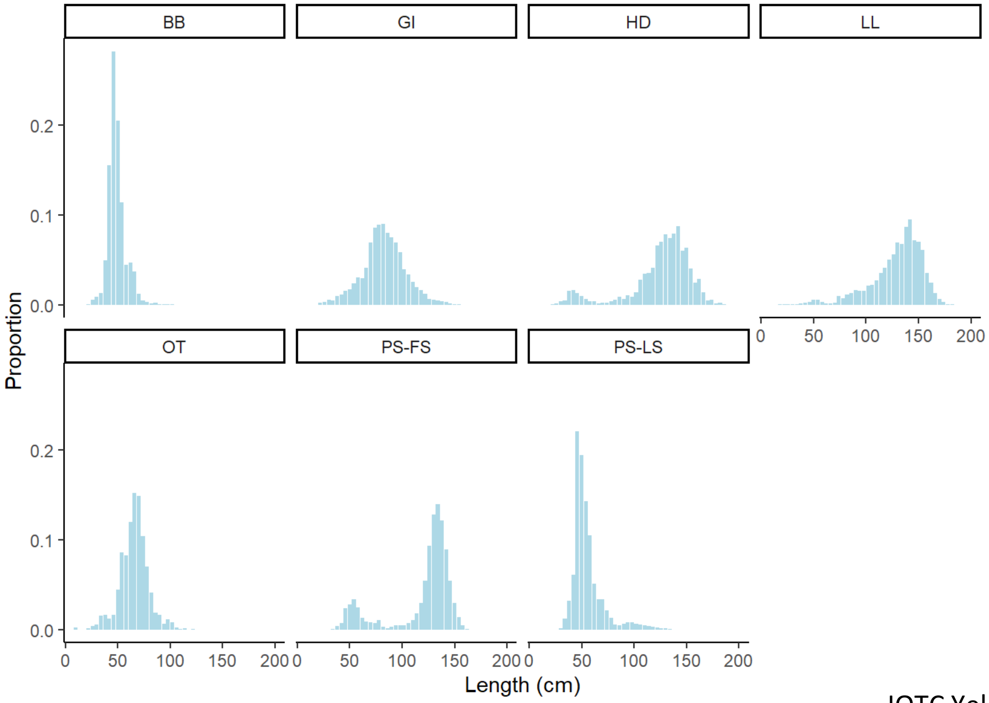


Normal + Logistic

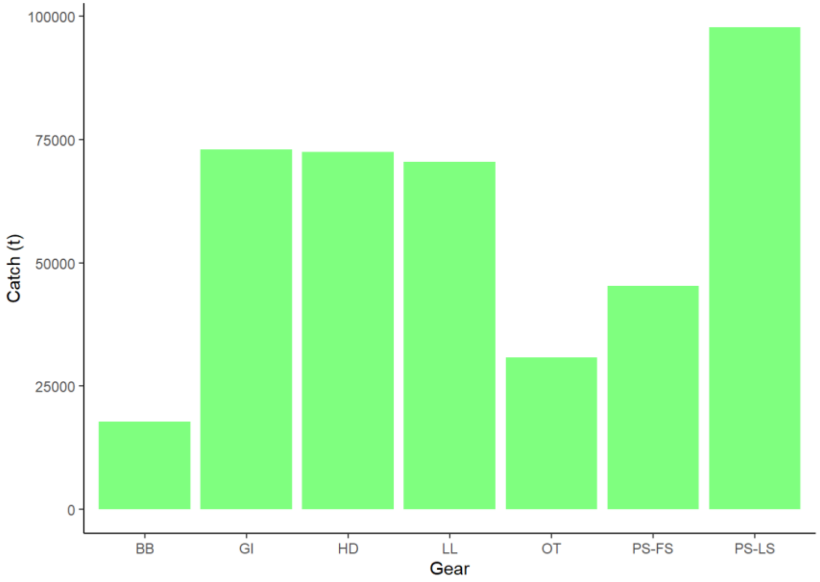


ILLUSTRATIVE EXAMPLE: IO YELLOWFIN

Yellowfin Length Frequency Data + Catches

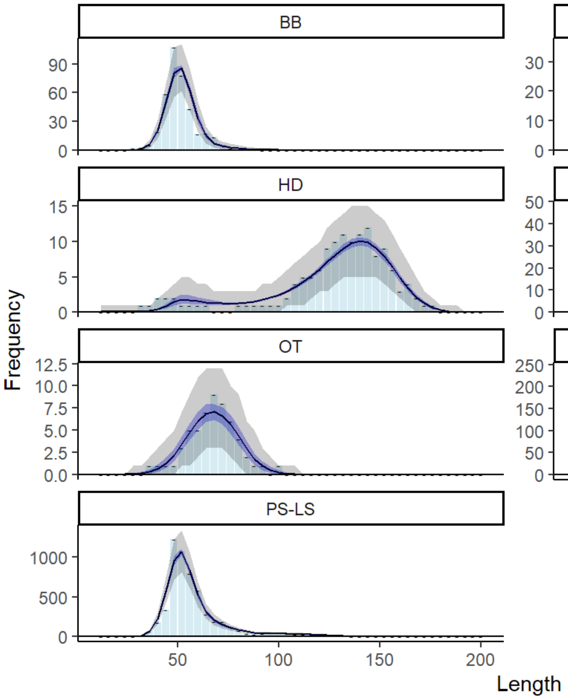


Aggregated 2014 – 2018



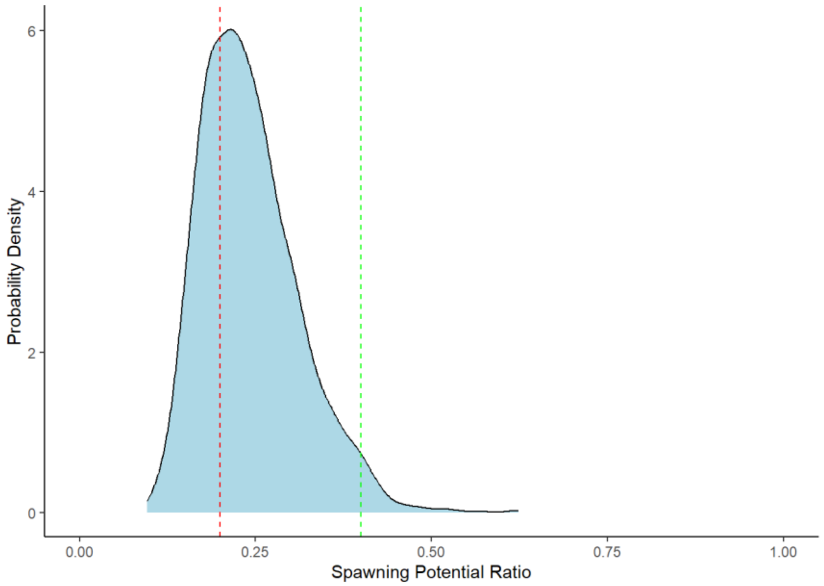
IOTC Yellowfin data used in the 2019 stock assessment SS3 V3.30
https://iotc.org/sites/default/files/documents/2019/09/IOTC-2019-WPTT21-DATA15-YFT_SA_0.zip#"Stock assessment inputs (SS3 and SCAA) for YFT"#"IOTC-2019-WPTT21-DATA15"

Fitted model

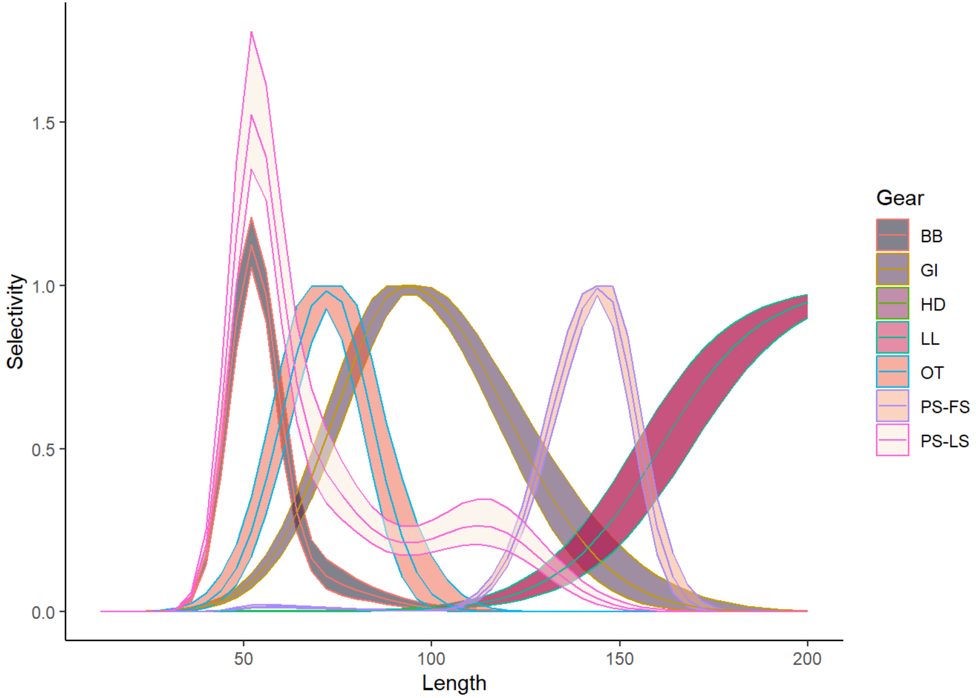


- Selectivity mixtures
- Length-inverse M
- Fishbase L_∞

Results



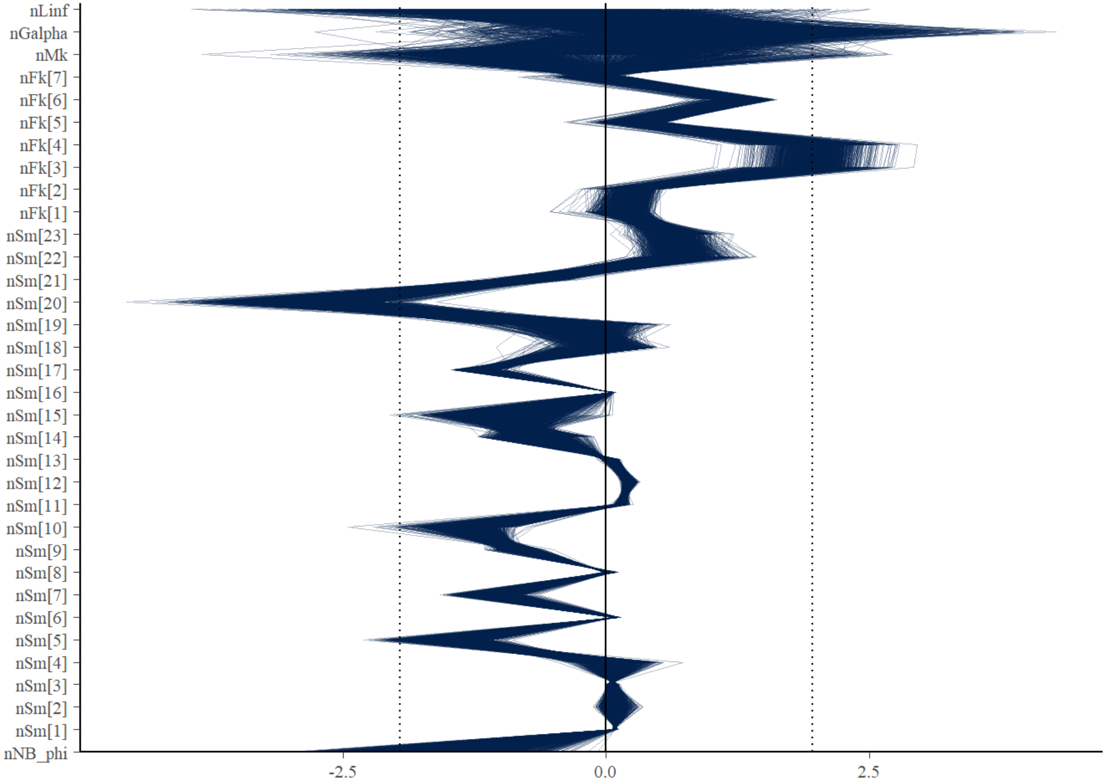
Probability Density
6
4
2
0
0.00
0.25
0.50
0.75
1.00
Spawning Potential Ratio



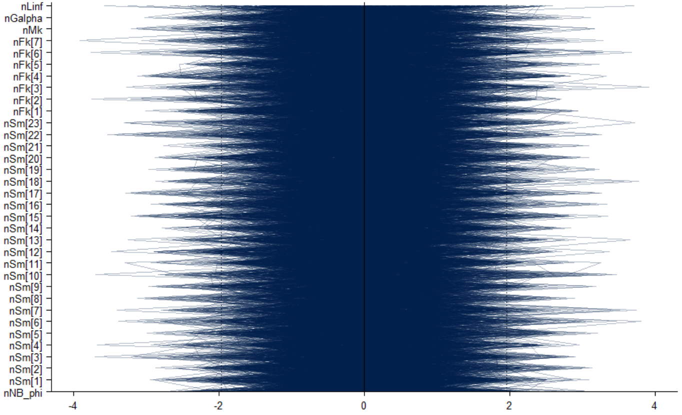
Selectivity
1.5
1.0
0.5
0.0
50
100
150
200
Length
Gear
BB
GI
HD
LL
OT
PS-FS
PS-LS

Parameter Estimates

Posterior



Prior



Conclusion

- Bayesian length-based catch curve with flexible modelling of mortality-at-length implemented in the *fishblicc* package
- *Key assumption* is population is in stationary state
- *Fits* single and multiple sample length frequency data
- *Estimates* F-at-length, selectivity, SPR, YPR etc.
- *Used* for data limited assessments + examine selectivity models
- *Caution* with overfitting (use sensitivities)