Please do not cite this document

until the final version of the software is completed

Your comments for improvement are welcome.

AD Model Builder Implementation of ASPM - Age-structured production model

- Final stage of the ADMB based ASPM software development project (2008-2010) -

October, 2009

Rebecca Rademeyer
University of Cape Town, South Africa

Tom Nishida*
National Research Institute of Far Seas Fisheries (NRIFSF), Japan
tnishida@affrc.go.jp

* Corresponding author

Contents

Part 1 : Outline & Concepts (PowerPoint) ---------- 03-06
Part 2 : User's Guide
1. HOW TO RUN ASPM---------- 08
2. INPUT FILES
2.1 Control file---------- 09-
2.2 *Parameter guesses* file---------- 10
2.3 *Biological data* file---------- 10
2.4 Index file---------- 11
2.5 Fishery file---------- 11
2.6 Projection file---------- 12
3. RUNNING MCMCs---------- 12
4. OUTPUT FILES---------- 13
5. Suggestions---------- 14
Part 3 : MODEL EQUATIONS---------- 16-21

This document is submitted to the IOTC WPTT11 (2009) (Oct., 15-23, 2009) in Mombasa, Kenya

Part 1

Outline & Concepts (PowerPoint)

Please do not cite this document
until the final version of the software is completed

Comparisons

	[CPUE] (YR)	[CPUE] (Q)	[SEX]	[CAS]	[CAA]	[AGE]	[SPACE]	[Tag/Move]	
PM	YES	NO	NO	NO	NO	NO	NO	NO	simple
ASPM	YES	NO	NO	NO	YES	YES	NO	NO	intermediate
IM	YES	YES	YES	YES	NO	YES	YES	YES	integrated

PM(Production models) : ASPIC, PRODFIT, PROCEAN etc
IM(integrated models): MFCL, SS3 etc

Models with more INFO (Yes), more realistic if INFO are robust & reliable.
In not there are more uncertainties. **Trade off**

Need to compare, cross-check & evaluate by 3 basic models

Initial ASPM software by Restrepo (ICCAT, 1997)

➔ IOTC YFT, BET & SWO SA (2000-2007)
➔ FORTAN based ASPM
outdated and limited capabilities

- **Very slow operating speed (esp., bootstrap)**
(story of my assistant)
- **(max.) 4 fleets**
- **Steepness (can't be fixed) 0.999 problem**
- **CAA: need to estimate <u>outside</u> (S-VPA)**

Background (ADMB-ASPM)

➔ 2008: approach several experts for re-code

➔ Original ASPM (Restrepo) : small pelagic SA
Punt + Butterworth (Univ. of Cape Town)(1992)

➔ introduction of Butterworth's staff
Rebecca Rademeyer
(expert on ADMB + Stat. model such ASPM)

➔ improve FORTAN-ASPM
add more functions (final stage) (2008-2010)

Comp FORTAN vs. ADMB ASPM(I)

	Fortran	ADMB
Fleets	**4**	**no limits**
Speed	**1**	**5-10 times faster**
Catch eq.	**Pope approx.**	**Baranov eq.**
Minimization routine	**simplex**	**Automatic differentiation**

Comp. FORTAN vs. ADMB ASPM(II)

	Fortran	ADMB
Uncertain (SE)	**bootstrap**	**delta method & MCMC**
Selectivity (CAA)	**Outside (e.g. S-VPA)**	***can be*** *estimated* ***within the model*** or can be fixed.
Projection	**deterministic**	**stochastic [catch/F scenario]**

Global evaluations by 3 basic models

(1) Simple : PM (ASPIC, PROOCEAN etc)
(2) Inter-mediate : INTM-M (ASPM, ASIA etc)
(3) Integrated : INTG-M (MFCL, SS3 etc)

Can not live with only one model

Important : **Global comp, cross-check & evaluation**

[PM] ↔ [INTM-M] ↔ [INTG-M]

(good example :last SWO SA)

Why ? Can not live with only one model

	[CPUE] (YR)	[CPUE] (Q)	[SEX]	[CAS]	[CAA]	[AGE]	[SPACE]
PM	YES	NO	NO	NO	NO	NO	NO
INTM-M	YES	NO	YES/NO	NO	YES	YES	NO
INTG-M	YES	YES	YES	YES	NO	YES	YES

More INFO → **more realistic**

↕ Trade off

More Uncertainty [INFO (**YES**)] → More Uncertainty [results]

⇒ Global evaluations by 3 basic models

Please do not cite this document

until the final version of the software is completed

Part 2

Users Guide

1. HOW TO RUN ASPM

To run ASPM, you need six input files (see next section). One of these is the *control* file, which controls some aspects of the run such as problem dimensions, i.e., numbers of years, numbers of age classes, etc.

To run ASPM, type aspm at the DOS prompt where you should have all the INPUT files as below:

C:\ (folder of your ASPM INPUT files) > aspm

The results will be sent to a series of output files which are described in the next section To run with starting values other than those in the *parameter guesses* file (ASPM.pin, see below) (for example in a file named otherInitial.par), type

C:\ (folder of your ASPM INPUT files) >aspm –ainp otherInitial.par

2. INPUT FILES

Six input files are required, their names cannot be changed:

Control file
Parameter guesses file
Biological data file
Index file
Fishery file
Projection file

In all these files, "#" precedes a comment line. In all the input files, extra comment lines can be added without affecting the run, as long as these line starts with "#". The example files contain annotations for input help. Their contents are as follows:

2.1 Control File (named: control.inp)

Line	Entry
1-3	Comments
4	First year of catch data
5	Last year of catch data
6	Comment
7	Number of fisheries or gear types for which yield data are available
8	Comment
9	Number of ages (last one is a plus group)
10	Comment
11	Stock-recruitment curve (1= Beverton-Holt, other = Ricker).
12	Comment
13	First year with recruitment fluctuations (see equation (21))
14	Last year with recruitment fluctuations
15	Comment
16	Standard deviation (σ_R) for the stock recruitment fluctuations (see equations (4) and (21))
17	Comment
18	Deterministic (=-1) or stochastic recruitment (>0)
19	Comment
20	Phase for dummy parameter. Always -1 except if running with ALL the parameters fixed, in that case it needs to be >0.
21	Comment
22	Phase for estimation of virgin spawning biomass (K^{sp}), i.e. whether virgin biomass is estimated (>0) or fixed (to value read in aspm.pin) (-1)
23	Comment
24	Phase for estimation of steepness (h), i.e. whether steepness is estimated (>0) or fixed (to value read in aspm.pin) (-1)
25	Comment
26	Phase for estimation of initial biomass as a fraction of virgin biomass (θ, see equation (8)), i.e. whether θ is estimated (>0) or fixed (to value read in aspm.pin) (-1)
27	Comment
28**	Phase for estimation of deviation from equilibrium age-structure* in the first year (ϕ, see equation (11)), i.e. whether ϕ is estimated (>0) or fixed (to value read in aspm.pin) (-1)
29	Comment
30	Phase for estimation of selectivity, i.e. whether selectivity is estimated (>0) or fixed (-1) (to value read in aspm.pin)
31	Comment
33	Type of weighting for the CPUE indices: 1 = maximum likelihood, 2 = equal weights, -1 = inverse-variance weighting for each point.

<u>Note</u>

The inputs corresponding to “phases” (lines 22, 24, 26 and 28) allow the minimization to be carried out over a subset of the parameters, while the others are fixed. In a non-linear model it can be useful to estimate the different parameters during different phase.

* Equilibrium age-structure:
Proportion of fish in each age class in the population before any fishing occurred

** If you want to start your assessment in 1970, for example, but you know that some catch was taken before, you can't assume the equilibrium age-structure (from your previous question). The phi is similar to a fishing mortality, a high phi will tend to give you a age-structure with less older fish than at equilibrium (phi=0). The "phase" is at what stage phi is going to be estimated - if the phase is negative, then phi is going to be fixed to the value in aspm.pin (you'd probably want phi=0 in that case).

2.2 Parameter guesses File (named: aspm.pin)

Line	Entry
1	Comment
2	Guess for ln of virgin spawning biomass ($\ln(SSB_0)$)
3	Comment
4	Guess for steepness
5	Comment
6	Guess for θ
7	Comment
8	Guess for ϕ
9	Comment
10	Comment
11-i	Guess for commercial selectivities
i+1	Comment
i+2-ii	Guesses for recruitment deviations

Notes

If you want the biomass at the beginning of the first year to be the virgin biomass, line 6 must be 1 and line 8 must be zero, with the phase parameters for θ and ϕ (lines 26 and 28 of the control.inp file must be negative).

Guesses for the selectivities go in the order: fleet, periods, ages (minus to plus). e.g. two fleets, the first fleet has two selectivity periods and age minus=0, age plus=5, second fleet has only one period and goes from age minus=2 to age plus=7:

# 0	1	2	3	4	5	6	7	
0.1	0.2	0.3	1.0	0.6	0.3			# fleet1, period 1
0.15	0.3	0.4	1.0	0.5	0.2			# fleet1, period 2
		0.6	0.8	0.9	1.0	0.8	0.6	# fleet2, period 1

Guesses for recruitment deviations are not really necessary.

2.3 Biological Data File (named: biological.inp)

Line	Entry
1-5	Comments
6	Weights at age at the start of the year for all ages
7-9	Comments
10	Weights at age at the middle of the year
11-14	Comments
15	Natural mortality at age
16-18	Comments
19	Proportion maturity at age

2.4 Index File (named: Index.inp)

Line	Entry
1-3	Comments
4	Number of indices
5	Comments
6	Index type for each index (1 = biomass, other = numbers).
7	Comments
8	Index timing: Month of the year or -1 for mid-year.
9	Comments
10	Fleet the index refer to
11-13	Comments
14..	Nyears: Year, index value (0 for missing), CV(index), selectivity of the index at age.
…	start again for next index

<u>Note</u>

The CV(index) values will be ignored if ML or equal weighting options are selected in the control file. The indices must start from the begging year of your data set, if index doesn't exist for a particular year, set to zero.

2.5 Fishery File (named: fishery.inp)

Line	Entry
1-4	Comments
5	Number of selectivity periods for each fleet
6	Comments
	For each fleet
	Comments
	First year and last year for each selectivity period
x	Comments
x	Minus group for each fleet
x	Comments
x	Plus group for each fleet
x	Comments
x	Weight given to each commercial catch-at-age data set in the likelihood (for each fleet)
x	Comments
x	for each year: year, catch for each fleet
	For each fleet
x	Comments
x	for each year: catch-at-age for each age

<u>Note</u>

If the selectivities are fixed (line 30 in control.inp is set to -1), then have the minus groups for each fleet be 0 and the plus groups for each fleet be the maximum age.

If the model is not fitted to the catch-at-age information (CAA weight=0), the program will still compare the observed and predicted CAA (but it wont be included in the likelihood). So if you don't have CAA information, rather fill the matrix with zeros.

2.6 Projections File (named: projection.inp)

Line	Entry
1-3	Comments
4	Number of years to project forward
5	Comments
6	Project with constant catch (=1) or constant F (=2)
7	Comments
8	Future catch by fleet if project with constant catch
9	Comments
10	Future F by fleet if project with constant F

Note

For each fleet, selectivity in the future is assumed to be the average of the last 5 years.

In a maximum likelihood run, the projections are deterministic, i.e. the recruitment is exactly determined by the stock-recruitment curve. If MCMC are run however, variations around the stock-recruitment curves are generated for each year, i.e. stochastic projections.

3. RUNNING MCMCs

ADMB includes a Markov Chain Monte Carlo (MCMC) routine for Bayesian analysis. The objective is to represent the posterior distributions by means of a (large) number of vectors of parameters. The basic idea is to set up a (long) "chain" which starts at a pre-specified parameter vector and that then traverses the posterior distribution. The contribution from equation 21 correspond to a prior on the distribution of the recruitment residuals, while priors on the other estimable parameters (SSB_0, θ, ϕ and the selectivity parameters) are taken to be uniform over wide and/or feasible ranges with the intent that they be uninformative. The initial parameter vector used to start the MCMC computational process is the mode of the posterior. A chain of N iterations is run and the chain is "thinned" by taking every m^{th} value in the chain. The results of the first iterations (5-50% of the total chain length) should be discarded to allow for a "burn-in" period, i.e. reduce the impact of the initial parameter vector.

To run MCMCs, type aspm –mcmc N –mcsave m as below:

C:\ (folder of your ASPM INPUT files) > aspm –mcmc N –mcsave m

where N is the number of simulations performed and every m'th simulations are saved. To get the desired output, type aspm –meval as below:

C:\ (folder of your ASPM INPUT files) >aspm –meval

See the ADMB manual for further MCMC options.

The output file ASPM.hst contains the means, standard deviations and observed distributions for all parameters included in the aspm.std file (after the estimable parameters, i.e. starting from K downwards). Once MCMC for a particular model has been run, stochastic projections can be run for different future catches (or F) without rerunning the MCMC, just changing the future catches and rerun 'aspm –meval'.

4. OUTPUT FILES

ASPM creates a series of output files as below. Most of them are standard ADMB output files.

ASPM.rep

Results for the run, including spawning biomass, numbers-at-age, recruitment, fits to the data, fishing mortality, MSY and related quantities, etc. "Bexp" refers to the exploitable biomass (see equation 7). "Fmort" is the fishing mortality.

proj.out

Projected spawning biomass and projected SSB_{MSY}.

ASPM.par

A standard ADMB output file, giving the objective function value, its gradient (this should be very small if the model has converged) and the parameters estimated/fixed for that run.

ASPM.std

A standard ADMB output file, with the parameters estimated for that run and their estimated Hessian-based standard deviation. Annual SSB, Btot, SSB/SSBmsy, F, F/Fmsy and MSY their estimated standard deviations are also included.

ASPM.cor

A standard ADMB output file, i.e., Correlations of the parameter estimates.

ASPM.hst

A standard ADMB output file when MCMCs are run (see above).

MCMC_B.csv

SSB, Btot and SSB/SSBmsy for each year. If MCMC has been run, one vector for each iteration. Probability intervals for these parameters can then be computed, taken the burn-in period into account (see above).

MCMC_F.csv

F, F/Fmsy and MSY for each year. If MCMC has been run, one vector for each iteration. Probability intervals for these parameters can then be computed, taken the burn-in period into account (see above).

5. Suggestions

- Always try different starting guesses to see if the program converges on the same solution.

  If the model does not converge, you can try restarting it with the parameter that have just been estimated, to do this type ASPM –ainp aspm.par as below;

  C:\ (folder of your ASPM INPUT files) > ASPM –ainp aspm.par

  i.e. it will use the values in aspm.par as initial guesses.

- Since the input and output file names are fixed, it's easier to keep each run in a separate folder.

Part 3

Model equations

Age-Structured Production Model Methodology

The model detailed is an Age-Structured Production Model (ASPM) (e.g. Hilborn, 1990). Models of this type fall within the more general class of Statistical Catch-at-Age Analyses. The approach used in an ASPM assessment involves constructing an age-structured model of the population dynamics and fitting it to the available abundance indices by maximising the likelihood function. The model equations and the general specifications of the model are described below, followed by details of the contributions to the (penalised) log-likelihood function from the different sources of data available and assumptions concerning the stock-recruitment relationship. Quasi-Newton minimization is used to minimize the total negative log-likelihood function (the package AD Model Builder[TM], Otter Research, Ltd is used for this purpose).

1. Population dynamics

1.1 Numbers-at-age

The resource dynamics are modelled by the following set of population dynamics equations:

$$N_{y+1,1} = R_{y+1} \qquad (1)$$

$$N_{y+1,a+1} = \left(N_{y,a}\, e^{-M_a/2} - \sum_f C^f_{y,a} \right) e^{-M_a/2} \qquad \text{for } 1 \le a \le m-2 \qquad (2)$$

$$N_{y+1,m} = \left(N_{y,m-1}\, e^{-M_{m-1}/2} - \sum_f C^f_{y,m-1} \right) e^{-M_{m-1}/2} + \left(N_{y,m}\, e^{-M_m/2} - \sum_f C^f_{y,m} \right) e^{-M_m/2} \qquad (3)$$

where

$N_{y,a}$ is the number of fish of age *a* at the start of year *y* (which refers to a calendar year),

R_y is the recruitment (number of 0-year-old fish) at the start of year *y*,

M_a denotes the natural mortality rate for fish of age *a*,

$C^f_{y,a}$ is the predicted number of fish of age *a* caught in year *y* by fleet *f*, and

m is the maximum age considered (taken to be a plus-group).

These equations reflect Pope's form of the catch equation (Pope, 1972) (the catches are assumed to be taken as a pulse in the middle of the year) rather than the more customary Baranov form (Baranov, 1918) (for which catches are incorporated under the assumption of steady continuous fishing mortality). Pope's form has been used in order to simplify computations. As long as mortality rates are not too high, the differences between the Baranov and Pope formulations will be minimal.

1.2. Recruitment

The number of recruits at the start of year *y* is assumed to be related to the spawning stock size (i.e. the biomass of mature fish) by a Beverton-Holt stock-recruitment relationship (Beverton and Holt, 1957), parameterised in terms of the "steepness" of the stock-recruitment relationship, *h*, and the pre-exploitation equilibrium spawning biomass, SSB_0 , and recruitment, R_0 and allowing for annual fluctuation about the deterministic relationship:

$$R_y = \frac{4hR_0 SSB_y}{SSB_0(1-h)+(5h-1)SSB_y} e^{(\varsigma_y - \sigma_R^2/2)} \qquad (4)$$

where

ς_y reflects fluctuation about the expected recruitment for year *y*, which is assumed to be normally distributed with standard deviation σ_R (which is input in the applications considered here); these residuals are treated as estimable parameters in the model fitting process.

SSB_y is the spawning biomass at the start of year *y*, computed as:

$$SSB_y = \sum_{a=1}^{m} f_{y,a} w_{y,a}^{strt} N_{y,a} \qquad (5)$$

where

$w_{y,a}^{strt}$ is the mass of fish of age *a* during spawning, and

$f_{y,a}$ is the proportion of fish of age *a* that are mature.

In the fitting procedure, SSB_0 is estimated while *h* can be estimated or fixed.

1.3. Total catch and catches-at-age

The fleet-disaggregated catch by mass in year *y* is given by:

$$C_y^f = \sum_{a=1}^{m} w_{y,a}^{mid} C_{y,a}^f = \sum_{a=1}^{m} w_{y,a}^{mid} N_{y,a} e^{-M_a/2} S_{y,a}^f F_y^f \qquad (6)$$

where

$w_{y,a}^{mid}$ denotes the mass of fish of age *a* landed in year *y*,

$C_{y,a}^f$ is the catch-at-age, i.e. the number of fish of age *a*, caught in year *y* by fleet *f*,

$S_{y,a}^f$ is the commercial selectivity of fleet *f* (i.e. combination of availability and vulnerability to fishing gear) at age *a* for year *y*; when $S_{y,a}$ = 1, the age-class *a* is said to be fully selected, and

F_y^f is the proportion of a fully selected age class that is fished, for fleet *f*.

The model estimate of the mid-year exploitable ("available") component of biomass is calculated by converting the numbers-at-age into mid-year mass-at-age (using the individual weights of the landed fish) and applying natural and fishing mortality for half the year:

$$B_y^f = \sum_{a=1}^{m} w_{y,a}^{mid} S_{y,a}^f N_{y,a} e^{-M_a/2}(1-\sum_f S_{y,a}^f F_y^f/2) \qquad (7)$$

1.4. Initial conditions

For the first year (y_0) considered in the model, the stock is assumed to be at a fraction (θ) of its pre-exploitation biomass, i.e.:

$$SSB_{y_0} = \theta \cdot SSB_0 \qquad (8)$$

with the starting age structure:

$$N_{y_0,a} = R_{start} N_{start,a} \qquad \text{for} \quad 1 \le a \le m \qquad (9)$$

where

$$N_{start,1} = 1 \qquad (10)$$

$$N_{start,a} = N_{start,a-1} e^{-M_{a-1}}(1-\phi S_{a-1}) \qquad \text{for} \quad 2 \le a \le m-1 \qquad (11)$$

$$N_{start,m} = N_{start,m-1} e^{-M_{m-1}}(1-\phi S_{m-1}) \Big/ (1-e^{-M_m}(1-\phi S_m)) \qquad (12)$$

where ϕ characterises the average fishing proportion over the years immediately preceding y_0.

2. The (penalised) likelihood function

The model can be fit to CPUE and commercial catch-at-age data to estimate model parameters (which may include residuals about the stock-recruitment function, through the incorporation of a penalty function described below). Contributions by each of these to the negative of the (penalised) log-likelihood (- $\ell\text{n}L$) are as follows:

2.1 CPUE relative abundance data

The likelihood is calculated assuming that an observed CPUE index for a particular fishing fleet is log-normally distributed about its expected value:

$$I_y^i = \hat{I}_y^i \exp(\varepsilon_y^i) \quad \text{or} \quad \varepsilon_y^i = \ell\text{n}(I_y^i) - \ell\text{n}(\hat{I}_y^i) \qquad (13)$$

where

I_y^i is the CPUE index for year y and series i,

$\hat{I}_y^i = \hat{q}^i \hat{B}_y^f$ is the corresponding model estimate, where $\hat{B}_y^f$ is the model estimate of exploitable resource biomass, given by equation (7),

$\hat{q}^i$ is the constant of proportionality (catchability) for CPUE series i, and

ε_y^i from $N\left(0,(\sigma_y^i)^2\right)$.

The contribution of the CPUE data to the negative of the log-likelihood function (after removal of constants) is then given by:

$$-\ell\text{n}\, L^{CPUE} = \sum_i \sum_y \left[\ell\text{n}\left(\sigma_y^i\right) + \left(\varepsilon_y^i\right)^2 / 2\left(\sigma_y^i\right)^2 \right] \quad (14)$$

where

σ_y^i is the standard deviation of the residuals for the logarithm of index *i* in year *y*.

If all the values for all indices are given equal weight, the standard deviation can be set to:

$$\hat{\sigma}_y^i = \sqrt{\sum_i \left[\frac{1}{n_i} \sum_y \left(\ell\text{n}(I_y^i) - \ell\text{n}(q^i \hat{B}_y^f) \right)^2 \right]} \quad (15)$$

or, if all values within an index are to have equal weights but each index is weighted depending on how it is fitted by the model (maximum likelihood weighting) then:

$$\hat{\sigma}_y^i = \sqrt{\frac{1}{n_i} \sum_y \left(\ell\text{n}(I_y^i) - \ell\text{n}(q^i \hat{B}_y^f) \right)^2} \quad (16)$$

where

n_i is the number of data points for CPUE index *i*.

Alternatively, the variances could be input for each value, based on external information.

The catchability coefficient q^i for CPUE index *i* is estimated by its maximum likelihood value:

$$\ell n\, \hat{q}^i = 1/n_i \sum_y \left(\ln I_y^i - \ln \hat{B}_y^f \right) \quad (17)$$

2.3.Catches-at-age

The contribution of the catch-at-age data to the negative of the log-likelihood function under the assumption of an “adjusted” lognormal error distribution is given by:

$$-\ell\text{n}\, L^{CAA} = \sum_f \sum_y \sum_a \left[\ell n \left(\sigma_{com}^f / \sqrt{p_{y,a}^f} \right) + p_{y,a}^f \left(\ell n\, p_{y,a}^f - \ell n\, \hat{p}_{y,a}^f \right)^2 / 2\left(\sigma_{com}^f\right)^2 \right] \quad (18)$$

where

$p_{y,a}^f = C_{y,a}^f / \sum_{a'} C_{y,a'}^f$ is the observed proportion of fish caught in year *y* by fleet *f* that are of age *a*,

$\hat{p}_{y,a}^f = \hat{C}_{y,a}^f / \sum_{a'} \hat{C}_{y,a'}^f$ is the model-predicted proportion of fish caught in year *y* by fleet *f* that are of age *a*,

where

$$\hat{C}_{y,a}^f = N_{y,a}\, e^{-M_a/2}\, S_{y,a}^f\, F_y^f \quad (19)$$

and

σ^f_{com} is the standard deviation associated with the catch-at-age data of fleet *f*, which is estimated in the fitting procedure by:

$$\sigma^f_{com} = \sqrt{\sum_y \sum_a p^f_{y,a} \left(\ell n\, p^f_{y,a} - \ell n\, \hat{p}^f_{y,a} \right)^2 / \sum_y \sum_a 1} \qquad (20)$$

The log-normal error distribution underlying equation (18) is chosen on the grounds that (assuming no ageing error) variability is likely dominated by a combination of interannual variation in the distribution of fishing effort, and fluctuations (partly as a consequence of such variations) in selectivity-at-age, which suggests that the assumption of a constant coefficient of variation is appropriate. However, for ages poorly represented in the sample, sampling variability considerations must at some stage start to dominate the variance. To take this into account in a simple manner, motivated by binomial distribution properties, the observed proportions are used for weighting so that undue importance is not attached to data based upon a few samples only.

Commercial catches-at-age are incorporated in the likelihood function using equation (18), for which the summation over age *a* is taken from age a_{minus} (considered as a minus group) to a_{plus} (a plus group).

2.5. Stock-recruitment function residuals

The stock-recruitment residuals are assumed to be log-normally distributed. Thus, the contribution of the recruitment residuals to the negative of the (now penalised) log-likelihood function is given by:

$$-\ell n L^{SRpen} = \sum_{y=y1}^{y2} \left[\varepsilon_y^2 \big/ 2\sigma_R^2 \right] \qquad (21)$$

where

ε_y from $N\left(0,(\sigma_R)^2\right)$, which is estimated for year *y*1 to *y*2 (see equation (4)), and

σ_R is the standard deviation of the log-residuals, which is input.

3. Model parameters

3.1. Fishing selectivity-at-age:

The commercial fleet-specific fishing selectivity, S^f_a, is estimated directly for age ‘minus’ to age ‘plus’. The estimated decreases from ages minus+1 to minus and ages plus-1 to plus are assumed to continue exponentially to ages 0 and *m* (maximum age considered) respectively.

Time dependence may be incorporated into these specifications by estimating different selectivity parameters for specific time periods, so that $S^f_a \rightarrow S^f_{y,a}$.

4. Projections

The resource is projected forward assuming either a constant catch or constant *F* (input for each fleet). The fishing selectivities used in the future are assumed to be equal to the average of the last 5 years. In a maximum likelihood context, future recruitment is assumed to be given exactly by the stock-recruitment relationship ($\varepsilon_y = 0$, see equation 4) ; when running MCMC however, recruitment deviations are generated each year, ε_y from $N\left(0,(\sigma_R)^2\right)$.

REFERENCES

Baranov, F.T. 1918. On the question of the dynamics of the fishing industry. Nauchnyi issledovatelskii iktiologisheskii Institut Izvestia, I: 81–128.

Beverton, R.J.H., and Holt, S.J. 1957. On the dynamics of exploited fish populations. Fisheries Investment Series 2, Vol. 19, U.K. Ministry of Agriculture and Fisheries, London. 533pp.

Hilborn, R. 1990. Estimating the parameters of full age-structured models from catch and abundance data. International North Pacific Fisheries Commission Bulletin, 50: 207-213.

Pope, J.G., 1972. An investigation of the accuracy of Virtual Population Analysis using cohort analysis. International Commission for the North Atlantic Fisheries Research Bulleting, 9: 65–74