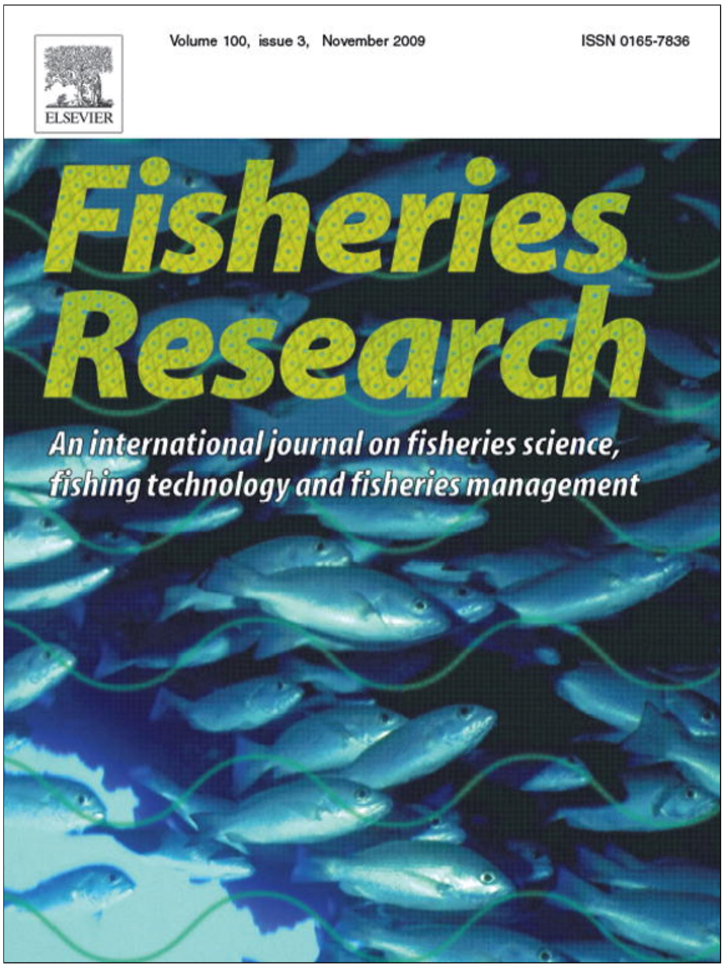
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Comparing three indices of catch per unit effort using Bayesian geostatistics

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ABSTRACT

In assessing a fish stock, indices based on catch per unit effort (CPUE) are frequently used. Estimates of three indices of catch per unit effort were compared here ($CPUE_1$, $CPUE_2$ and $CPUE_3$), considering the fitting of two models: (i) a bivariate geostatistical model for catch and effort; (ii) a bivariate model where catch and effort were considered spatially independent. For comparing the estimates of the three indices after the fitting of the two models, catch and effort data were simulated in different scenarios. The simulation study showed that, in general, the estimates of $CPUE_1$ expressed by the ratio of the means of catch and effort, present better results for different scenarios and that the estimates from (i) are better than (ii), mainly when there is a correlation between catch and effort and an additional spatial correlation.

1. Introduction

To evaluate a fish stock, data of catch and effort resulting from commercial fishing are usually used in heuristic relationships. Based on catch and effort data, indices of relative abundance are calculated in order to supply information about the stock. In a given inhabited area by a given stock, if the density (or concentration) of fish (biomass/volume) is constant for the whole area the CPUE is proportional to stock abundance (strict proportionality) (Clark, 1985). In some cases this relationship might not be linear. The examination of this relationship is not the main theme of this paper. However, in the light of this paper for any supposed model the relationship CPUE × abundance, it is necessary to estimate the CPUE in order to evaluate the abundance.

Detailed records, with information on the geographic coordinates where fishing occurred allow a spatial analysis of fishing. Normally a point of reference is given for each quadrat (sub-regions delimited by parallels and meridians) where fishing occurred (ICCAT, 2007).

Based on catch × effort data, three CPUE indices may be defined for a whole area:

$$CPUE_1 = \frac{1}{n}\sum_{i=1}^{n}\frac{C_i}{E_i}; \quad \text{mean of the ratios catch by effort;}$$

$$CPUE_2 = \frac{\sum_{i=1}^{n} C_i}{\sum_{i=1}^{n} E_i}; \quad \text{ratio of total catch by total effort;}$$

$$CPUE_3 = \frac{\sum_{i=1}^{n} C_iE_i}{\sum_{i=1}^{n} E_i^2}; \quad \text{ratio estimator}$$

as proposed by Snedecor and Cochran (1967),where $C_i, i = 1, 2, \ldots, n$, represents the catch in the ith quadrat and E_i the respective effort, n is the total number of quadrats superposed as an artificial grid in the fishing area.

The three indices may be described as C_i/E_i averages distinguished by the weighing criteria, that is $\sum_{i=1}^{n}(C_i/E_i)w_i$. In $CPUE_1$ the weighing factor is $w_i = 1/n$; in $CPUE_2$ it is $w_i = E_i/(\sum_{j=1}^{n} E_j)$ and in $CPUE_3$ we have $w_i = E_i^2/(\sum_{j=1}^{n} E_j^2)$.

Whenever C (Capture) be proportional to E (Effort), the regression line between them statistically goes through the origin, and can be fitted by the simple model $C_i = \beta E_i + \varepsilon_i$. $CPUE_3$, together with $CPUE_1$ and $CPUE_2$, are all unbiased estimates of the population ratio β in normally distributed populations. The choice among the three is a matter of precision: the most precise among the three is $CPUE_3$, $CPUE_2$ and $CPUE_1$ if the variance of ε (error term) is

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constant, proportional to E or to E^2, respectively. If the variance of ε increases moderately with E, CPUE$_2$ is still expected to perform well (Petrere Jr. et al., 2008). Data on catch and effort are usually not available for all quadrats to establish these indices, in other words, not all quadrats are fished. In this situation, a possibility of estimation is simply to use the observed data. This might not be the best option. According to Walters (2003), if a spatial correlation structure between catch rates is found, spatial statistics can be used for extrapolation to grid squares where no fishing took place, before calculating any abundance index. Moreover, this author claims that covariates such as water temperature can be used to provide estimates of the spatial trend. The oceanic surface temperature is an important covariate, as several studies show that it is correlated with CPUE (Dow et al., 1975; Dow, 1980; Evans et al., 1995; Fonteneau, 1995; Lima et al., 2000; Goodyear, 2003).

In this work a model was utilized for the variables catch and effort, whose covariance structure is described by a model of linear coregionalization (Gelfand et al., 2004), from now on this model will be called special bivariate model (SBM). This was one of the models investigated in the spatial analysis of catch and effort data. The choice was based on observations of Walters (2003) on the use of spatial statistical techniques and covariates and also taking into consideration that in practice the observed data are bivariate (catch and effort) (ICCAT, 2007), besides other characteristics that are inherent to certain fishing data sets, e.g., the existence of spatial correlation (Swain and Wade, 2004; Walters, 2003), relationship between catch and effort and that the effort can be considered random since it may depend, for example, on the commercial value of the target species, time of year, climate conditions, sea surface temperature, perception of fishermen of a fish stock (observation or non-observation), information from other fishermen (Sanchéz et al., 2004; Walters and Martell, 2004; Hilborn and Walters, 1992). Besides describing the structure of covariance between variables catch and effort and the spatial correlation, the model SBM shapes the structure of the cross covariance, i.e., the covariance between the effort at any location s_i and catch at location s_j, and vice versa. In other words, the observations (E_i, C_i) are treated as a sample of a bivariate spatial process. A fitting of the proposed model makes the extrapolation of catch and effort possible to quadrats that were not observed. Besides the linear coregionalization model, it was fitted a bivariate model was fitted in which catch and effort are considered spatially independent, from now on called bivariate model without spatial component (BMWSC).

Since the CPUE indices are used in the assessment of fish stocks it is important to assess the performance of the three indices (CPUE$_1$, CPUE$_2$ or CPUE$_3$) in different scenarios. Above all, it is essential to use methods that estimate each index accurately. Petrere Jr. et al. (2008) conducted a simulation study to compare CPUE$_1$, CPUE$_2$ and CPUE$_3$. However, these indices were not studied in the presence of spatial correlation.

We suggest that the indices CPUE$_1$, CPUE$_2$ and CPUE$_3$ be estimated as follows: by extrapolation of catch and effort to unfished quadrats; for this purpose one of the above models was adjusted, according to the Bayesian approach; after the extrapolation the indices CPUE$_1$, CPUE$_2$ and CPUE$_3$ were estimated based on the data set consisting of observed and predicted values.

The objectives of this study were:

(i) to compare the statistical behavior of the estimates of three indices (CPUE$_1$, CPUE$_2$ and CPUE$_3$), estimated when using the SBM model, based on simulated data sets of different scenarios;
(ii) to compare the estimates calculated through the interpolation of catch and effort in those not observed quadrats using the SBM and BMWSC models.

2. Materials and methods

The geostatistical techniques used here assume that the variables to be modelled follow normal distribution. The distributions of variables catch and effort are generally asymmetrical and in many cases the logarithmic transformation is sufficient to correct the lack of normality (Abuabara, 1996; Sanchéz et al., 2004). To perform the simulation study it was therefore assumed that catch and effort follow normal distribution in the logarithmic scale. In the following we describe the utilized models in the simulation studies and the inference procedures.

2.1. Spacial bivariate model (SBM)

When using Gelfand et al. (2004) model, the catch and effort observations across a region are treated as a sample from a bivariate spatial process. The proposed model, easily interpretable and computer processable, creates a structure of flexible covariance, where the ranges (i.e., the distance beyond which there is practically no spatial correlation between data points) associated with the variate are not necessarily the same. The authors show that there is an equivalence, based on reparametrization of the conditional specification given by Eq. (1) and the unconditional specification of the model.

Clearly there is a cause/effect between effort /catch. So when conditioning the model effort comes first, then capture. So, the logarithm of the fishing effort (Y_1) is modelled first and then the logarithm of catch, given by the logarithm of effort:

$$\begin{aligned} Y_1(s) &= \beta_{01} + \beta_{11}temp(s) + \sigma_1 w_1(s) \\ Y_2(s)\,\big|\,Y_1(s) &= \beta_{02} + \beta_{12}temp(s) + \alpha Y_1(s) + \sigma_2 w_2(s) + \tau_2 u_2(s), \end{aligned} \tag{1}$$

where $temp(s)$ represents the temperature at location s, $w_1(s)$ and $w_2(s)$ are Gaussian spatial processes with mean zero and variance 1, independent, but not identically distributed, and $u_2(s)$ has distribution $N(0,1)$. The term $\beta_{01}+\beta_{11}temp(s)$ in Eq. (1) determines the expected value of the logarithm of effort for a location s and $\beta_{02}+\beta_{12}temp(s)+\alpha(\beta_{01}+\beta_{11}temp(s))$, determines the expected value of the logarithm of capture. $\sigma_1 w_1(s)$ and $\alpha\sigma_1 w_1(s)+\sigma_2 w_2(s)$ accounts for spatial correlation in these quantities (effort and catch, respectively). The term $\tau_2 u_2(s)$ is responsible for microescale variation (nugget effect).The adopted correlation function was the exponential $\rho(d)$=exp$(-\phi d)$, with parameter ϕ_1 for Y_1 and ϕ_2 for Y_2, where d is the distance between any two points s, s'. The parameter ϕ, expresses how quickly the correlation drops to zero.

The use of conditional specification; the model for $Y_1(s)$ must not have a white noise component to ensure equivalence. The model written in its conditional form may have a pure nugget effect in the first equation. This occurs when spatial correlation is practically null, albeit in its presence, that is, when the correlation function parameter of the first equation (effort equation) is positive, we have for this equation a pure spatial effect. The remaining variation is inherited from the second equation, as in this equation $Y_2(s)$ is written in function of $Y_1(s)$.

Considering the model in its conditional form we have a large computational advantage, since instead of dealing with one covariance matrix $2n \times 2n$, two covariance matrices $n \times n$ are used (Gelfand et al., 2004).

Given any location s, it may be shown that the correlation between $Y_1(s)$ (logarithm of effort) and $Y_2(s)$ (logarithm of catch) is given by

$$\rho_{Y_1,Y_2} = \frac{\alpha\sigma_1^2}{\sqrt{\sigma_1^2(\alpha^2\sigma_1^2+\sigma_2^2)}} \tag{2}$$

in which $\alpha\sigma_1^2$ is the covariance between $Y_1(s)$ and $Y_2(s)$, σ_1^2 is the variance of $Y_1(s)$ and $\alpha^2\sigma_1^2+\sigma_2^2$ is the variance of $Y_2(s)$.

2.2. Bivariate model without spatial component (BMWSC)

In the simulation study besides SBM the BMWSC described in Eq. (3), which considers catch and effort as correlated with each other, but independent in space. This model was adjusted to determine the effect on the index estimates of $CPUE_1$, $CPUE_2$ and $CPUE_3$, when the spatial correlation is not considered.

$$\begin{aligned} Y_1(s) &= \beta'_{01} + \beta'_{11}temp(s) + \sigma'_1 u_1(s) \\ Y_2(s)|Y_1(s) &= \beta'_{02} + \beta'_{12}temp(s) + \alpha' Y_1(s) + \tau'_2 u_2(s) \end{aligned} \tag{3}$$

where $u_1(s)$ and $u_2(s)$ are independent and follow distribution $N(0,1)$.

Model of Eq. (3) is a particular case of model of Eq. (1), i.e., taking the correlation function $\rho(s-s') = \begin{cases} 0 \text{ if } s \neq s' \\ 1 \text{ if } s = s' \end{cases}$ for process $Y_1(s)$ in model of Eq. (1) with $\sigma_2 = 0$ leads to model of Eq. (3).

2.3. Inference procedure

The inference procedure which will be presented was utilized in both models (SBM and BMWSC). But it shall be pointed out that the correlation function parameters is ignored for BMWSC.

2.3.1. Prior distributions for the parameters

When using the Bayesian approach the incertainty of the unknown quantities in the model must be specified, this can be done by choosing a *prior* distribution for vector $\boldsymbol{\theta}$ of parameters of the model must be specified. These distributions possibly depend of other parameters, but as they are not the main model parameters they are called hyper-parameters. Banerjee et al. (2004) suggested the use of informative *priors* for the parameters σ^2 and ϕ. As it is usual in the literature, to the parameters σ_1^2, σ_2^2 and τ_1^2, inverted gamma *a prior* densities are attributed, that is, $\sigma_1^2 \sim IG(a_{\sigma_1}, b_{\sigma_1})$, $\sigma_2^2 \sim IG(a_{\sigma_2}, b_{\sigma_2})$ and $\tau_2^2 \sim IG(a_{\tau_2}, b_{\tau_2})$. For parameters ϕ_1 and ϕ_2, of the exponential correlation function, for the variables effort and catch, the same a *prior* gamma density was assumed, $\phi_1 \sim G(a_{\phi_1}, b_{\phi_1})$, $\phi_2 \sim G(a_{\phi_2}, b_{\phi_2})$ and, the hyper-parameters were obtained by resolving the equations $E(\phi_j) = 6/\max.dist = 0.05$ and $\mathrm{var}(\phi_j) = 20$, $j = 1, 2$ (large variance), where max.*dist* is the maximum distance between the observed locations, whose value is about 12. This *prior* reflects the fact that, as expected, for distances greater than $\max.dist/2$ the spatial correlation is approximately zero (Schmidt and Gelfand, 2003; Banerjee et al., 2004; Paez et al., 2005). To the parameter vectors $\boldsymbol{\beta}_1$ and $\boldsymbol{\beta}_2$ a normal *prior* density with covariance matrix of $\sigma_\beta^2 I$ was attributed where the σ_β^2 value was fixed at 100 (flat *prior*). The parametrization of the *prior* distributions used is this work is in agreement with Gelman et al. (2003).

2.3.2. Likelihood function

Considering the model in Eq. (1) the likelihood function can be written as the likelihood product for Y_1 and of likelihood for Y_2. The likelihood for Y_1 is given by

$$L(\boldsymbol{\beta}_1, \phi_1, \sigma_1^2|y_1) = (2\pi)^{-n/2}|\sigma_1^2\mathbf{R}_1|^{-1/2} \exp\left\{-\frac{1}{2}[y_1 - \mathbf{X}_1\boldsymbol{\beta}_1]^T[\sigma_1^2\mathbf{R}_1]^{-1}[y_1 - \mathbf{X}_1\boldsymbol{\beta}_1]\right\} \tag{4}$$

where $\mathbf{X}_1$ is the design matrix for Y_1, $\mathbf{X}_1$ is an $n \times 2$ matrix whose first column is composed by 1s and the second by $temp(s_i)$, $i = 1, 2, \ldots, n$, i.e., by the temperature readings in each quadrat. $\boldsymbol{\beta}_1 = [\beta_{01} \quad \beta_{11}]^T$ is the vector of coefficients of the covariates. $\boldsymbol{R}_1$ is a correlation matrix for the variate Y_1, and its elements are as $(\boldsymbol{R}_1)_{ii'} = \rho_1(s_i - s_{i'}) = \exp(-\phi_1|s_i - s_{i'}|)$.

The likelihood function for Y_2 is given by

$$L(\boldsymbol{\beta}_2, \phi_2, \sigma_2^2, \tau_2^2|\boldsymbol{y}_2, \boldsymbol{y}_1) = (2\pi)^{-n/2}|\sigma_2^2\boldsymbol{R}_2 + \tau_2^2\boldsymbol{I}|^{-1/2} \exp\left\{-\frac{1}{2}[\boldsymbol{y}_2 - \boldsymbol{X}_2\boldsymbol{\beta}_2]^T[\sigma_2^2\boldsymbol{R}_2 + \tau_2^2\boldsymbol{I}]^{-1}[\boldsymbol{y}_2 - \boldsymbol{X}_2\boldsymbol{\beta}_2]\right\} \tag{5}$$

where $\mathbf{X}_2$, $n \times 2$, is the design matrix for Y_2, identifies the $\mathbf{X}_1$, matrix and $\boldsymbol{\beta}_2 = [\beta_{02} \quad \beta_{12}]^T$ is the vector of coefficients of the covariates. $\mathbf{R}_2$ is the correlation matrix for the variable Y_2, where $(\mathbf{R}_2)_{ii'} = \rho_2(s_i - s_{i'}) = \exp(-\phi_2|s_i - s_{i'}|)$.

2.3.3. Posterior distributions for the parameters

In the Bayesian context inferences are based on the *posterior* distribution of the parameters. Due to the independence of the likelihoods given in Eqs. (4) and (5) and the *prior* parameters, the joint *a posterior* distribution for the parameters of model Eq. (1) is given by the product of the distributions

$$\pi(\boldsymbol{\beta}_1, \phi_1, \sigma_1^2|\boldsymbol{y}_1) \propto L(\boldsymbol{\beta}_1, \phi_1, \sigma_1^2|\boldsymbol{y}_1)p(\boldsymbol{\beta}_1)p(\phi_1)p(\sigma_1^2) \tag{6}$$

and

$$\pi(\boldsymbol{\beta}_2, \phi_2, \sigma_2^2, \tau_2^2|\boldsymbol{y}_2, \boldsymbol{y}_1) = L(\boldsymbol{\beta}_2, \phi_2, \sigma_2^2, \tau_2^2|\boldsymbol{y}_2, \boldsymbol{y}_1)p(\boldsymbol{\beta}_2)p(\phi_2)p(\sigma_2^2)p(\tau_2^2) \tag{7}$$

where the likelihood functions are given by Eqs. (4) and (5) and the *prior* distributions as described above.

Since the *posterior* distributions (6) and (7) do not have a definitive analytical form, methods MCMC (Gamerman and Lopes, 2006) were used to obtain a sample of the joint *a posterior* distribution of the parameters. More specifically, in these cases the Gibbs sampler with steps of Metropolis–Hastings can be used, since the complete conditional *posterior* distribution of some parameters is known and of others not. In Appendix A, the complete conditional *a posterior* distributions are shown for the parameters of model Eq. (1), required for the computational implementation of the algorithms MCMC.

Samples of the joint distribution were obtained using the computer program *WinBugs* (Spiegelhater et al., 2002). Chains (size 55,000) were used and the 5000 first samples were discarded (*burn-in*). Thereafter, the observations were stored 50 by 50 (*thinning*) to minimize problems of autocorrelation between samples. The convergence of the chains was verified by graphic trace analysis.

2.3.4. Predictive distribution

The prediction of future observations, based in the already observed values is done through the predictive distribution. Suposing that we want to make a prediction for a set of K places, $S_u = \{s_{u1}, s_{u2}, \ldots, s_{uK}\}$, one may form a vector of the variable in the predicted points $\mathbf{Y_u} = (Y(s_{u1}), Y(s_{u2}), \ldots, Y(s_{uK}))'$ and so obtaining the predictive distribution $(\boldsymbol{Y_u}|\boldsymbol{Y})$ given by:

$$p(\boldsymbol{Y_u}|\boldsymbol{Y}) = \int_\Theta p(\boldsymbol{Y_u}|\boldsymbol{Y}, \boldsymbol{\theta})p(\boldsymbol{\theta}|\boldsymbol{Y})d\boldsymbol{\theta} \tag{8}$$

in which $p(\mathbf{Y_u}|\mathbf{Y},\boldsymbol{\theta})$ is the conditional $\mathbf{Y_u}$ distribution given $\mathbf{Y}$ and $\boldsymbol{\theta}$, $p(\boldsymbol{\theta}|\mathbf{Y})$ is the *a posterior* distribution of $\boldsymbol{\theta}$, being $\boldsymbol{\theta}$ the parameters vector of the model associated to the variable which we want predict. In this context the integral of Eq. (8) has no analytic solution. So Monte Carlo methods (Gamerman and Lopes, 2006) may be employed in order to obtain samples from the predictive distribution. In this way effort and catch samples were obtained from their predictive distributions and the $CPUE_1$, $CPUE_2$ and $CPUE_3$ predictive distributions were obtained.

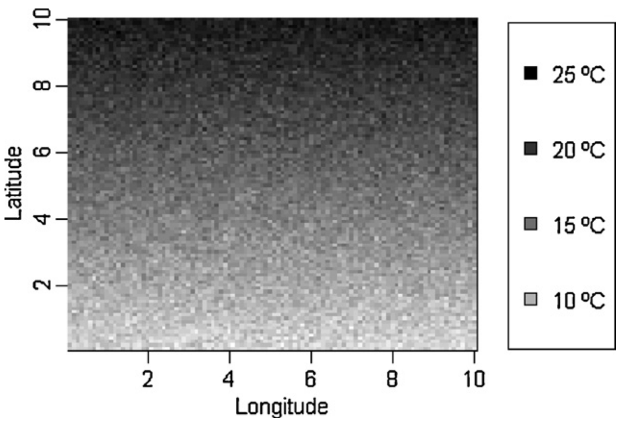

Fig. 1. Surface of temperature simulated values.

2.4. Simulation study

The three indices ($CPUE_1$, $CPUE_2$ and $CPUE_3$) were evaluated and the obtained estimates after the fitting of each model were compared in a simulation study. A grid was created with 100 regular points (10 × 10) to represent a (hypothetical) ocean region. At these points values for sea surface temperature (covariate) were simulated, considering that there is a gradient in the north–south direction. A gradient was assumed along which the temperature values increase according to the shift from south to north. Thus, we simulated temperature data in a range from 10 to 25 °C. In Fig. 1 it is shown the chart of the simulated temperature in the 10 × 10 grid.

The logarithm values of fishing effort (Y_1) and catch, given the logarithm of effort ($Y_2|Y_1$), were associated with the regular grid points. These values were directly simulated by the model SBM. When data of model SBM are simulated they generate data of catch and effort on a logarithmic scale, since here it is assumed that the data distribution of effort and catch is normal after a logarithmic transformation. To obtain simulated data of catch and effort it is therefore enough to apply the exponential function to the simulated data of model SBM. The software package *geoR* (Ribeiro and Diggle, 2001) of the statistical environment *R* (R Development Core Team, 2007) was used for data simulation.

When catch and effort data are collected, these variables may present different correlation intensities, and may also being or not being spatially correlated. So, depending on the intensity of the correlation between the two variables and of presence/absence of spatial correlation, we may have different sceneries for a data set. So the catch and fishing effort data were simulated in the following scenarios:

(a) considering a low correlation between the logarithm of catch and effort $\rho_{Y_1,Y_2} = 0.3$, reflecting a high uncertainty of the fisherman, and considering a low spatial correlation of the data, resulting in an effective range of 0.5 units of distance for effort (value solving the equation $\rho(d) = \exp(-\phi_1 d) = 0.05$) and 0.54 for catch (obtained by Newton's method, the equation of weighted correlation described by Gelfand et al. (2004)).
(b) considering a high correlation between catch and effort of $\rho_{Y_1,Y_2} = 0.8$, which reflects a less uncertainty of the fisherman, and considering the low spatial correlation of the data, which leads to an effective range of 0.5 units of distance for the effort and 0.52 for catch.
(c) considering a low correlation between catch and effort $\rho_{Y_1,Y_2} = 0.3$, and considering a strong spatial correlation of the data, which leads to an effective range of 6.67 units of distance for effort and 6.05 for catch.
(d) considering a high correlation between catch and effort of $\rho_{Y_1,Y_2} = 0.8$ and considering a strong spatial correlation of the data, which leads to an effective range of 6.67 units of distance for effort and 6.43 for catch.

Table 1
Parameters of the model SBM used to generate the data set in each scenario.

Scenario	Parameters									
	β_{01}	β_{11}	β_{02}	β_{12}	α	σ_1^2	σ_2^2	τ_2^2	ϕ_1	ϕ_2
(a)	2	0.25	0	0.35	0.28	2.3	1.8	0.1	6	5.5
(b)	2	0.25	0	0.35	1.2	2.3	1.8	0.1	6	5.5
(c)	2	0.25	0	0.15	0.28	2.3	1.8	0.1	0.45	0.5
(d)	2	0.25	0	0.15	1.2	2.3	1.8	0.1	0.45	0.5

In Table 1 the parameters used for simulating the data in each scenario are presented. Fig. 2 the graphs of the exponential correlation function are presented in two situations, considering a strong space correlation ($\phi = 0.5$) and a low spatial correlation ($\phi = 6$). In the first case (Fig. 2(a)) the correlation falls slowly with the increasing of the distance that is, the range of the spatial correlation is higher, while in the second case (Fig. 2(b)) the correction falls quickly.

In Fig. 3 are shown the relationships between the logarithm of effort and the logarithm of the catch for each simulated scenario.

A Thomas process (Reis, 1998) was then generated for the region under study (100-point grid), resulting in an aggregate spatial pattern, to represent the spatial distribution of locations of occurrence of fishing (Anganuzzi, 2004). For this purpose, the function *rThomas*() of the software package *spatstat* (Baddeley and Turner, 2005) of the statistical environment *R* (R Development Core Team, 2007) was used. The points generated by this process dropped in 85 quadrats whose central points were represented by "•" in Fig. 4. These 85 locations with temperature, catch and effort data were considered as observed data, which were used to adjust the model. In the 15 numbered locations the simulated data were omitted when fitting the models, in order to later on to predict the catch and effort values in these and so obtaining the estimates of the three indices $CPUE_1$, $CPUE_2$ and $CPUE_3$ using the predict values.

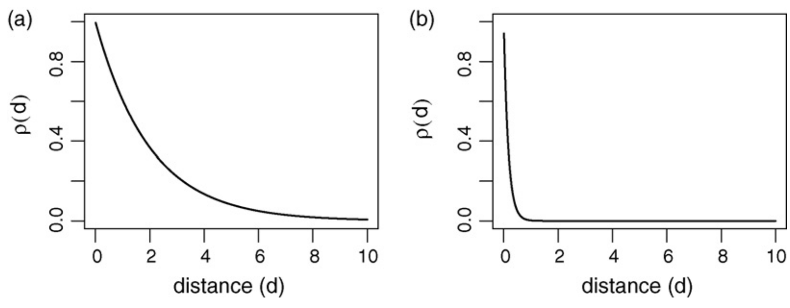

Fig. 2. Exponencial correlation function with para meters $\phi = 0.5$ (a) and $\phi = 6$ (b).

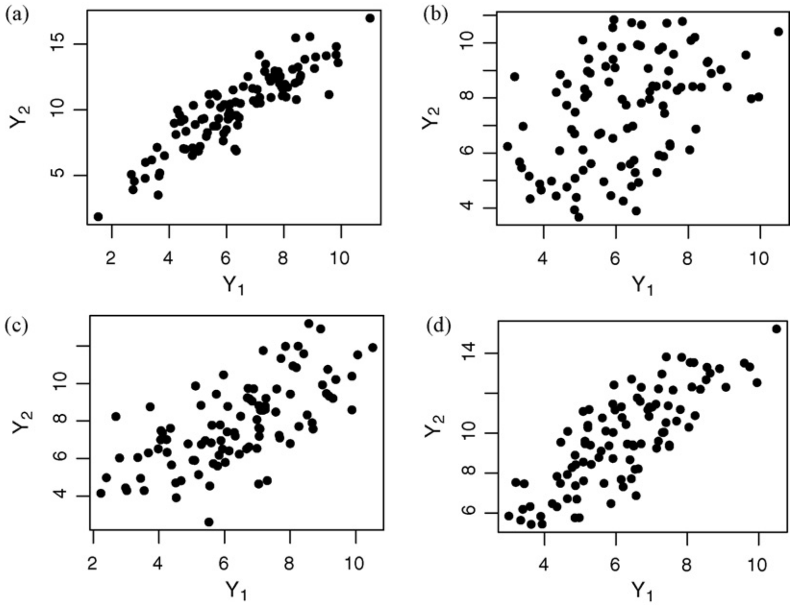

Fig. 3. Relationship between capture and effort for one realization of scenarios (a), (b), (c) and (d).

The values of $CPUE_1$, $CPUE_2$ and $CPUE_3$, where obtained from the simulated data set $(E(s_i),C(s_i))$, $i = 1, 2, \ldots, 100$. For each scenario (a)–(d) 50 data sets were simulated. In this way, 50 true values were obtained of $CPUE_1$, $CPUE_2$ and $CPUE_3$, for each scenario; that is, in the $\tilde{\imath}$th simulation it was obtained for the region of Fig. 4, $CPUE_{1l}$, $CPUE_{2l}$ e $CPUE_{3l}$, $l = 1,2, \ldots, 50$. For each of these sets, as previously described, 85 locations (Fig. 4) in the region were considered observed locations (measured locations as denominated in the geostatistical literature), and 15 locations were considered unobserved (unmeasured). For each of the data sets SBM and BMWSC models were adjusted. The program *WinBUGS* 1.4.3 was used to obtain the samples of the joint *posterior* parameters distribution of these models.

2.4.1. Estimation of the CPUE indices and the comparison criteria

After the adjustment of each model, the predictive distribution values of catch and effort were obtained for unobserved locations. Thereafter, the rates $CPUE_1$, $CPUE_2$ and $CPUE_3$ were estimated for each simulated data set. For obtaining the estimates it were considered the data set of "observed values" and values predicted at "unobserved locations".

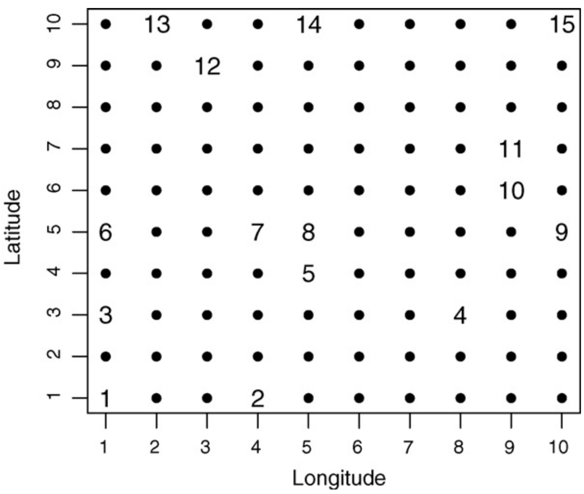

Fig. 4. Observed locations "•" and unobserved locations (numbered).

In the Bayesian approach, there is an *a posterior* sample for the indices, i.e, for each iteration of the MCMC method, an estimate is obtained for each of the indices $CPUE_1$, $CPUE_2$ and $CPUE_3$. To compare the estimates, a punctual estimative was considered, given by the median of the *posterior* sample.

To obtain an estimate for each index, a data set was composed of m pairs of catch and effort, derived from an observed sample and by K pairs of predicted catch and effort (for unobserved locations) by the adjusted model. The result was a set of $m + K = n$ pairs of catch and effort. Replacing this set in the formula of each index generates an estimate for each index. For example, $CPUE_{1\mathrm{est}} = (1/(m+K))\sum_{i=1}^{m+K} C_i/E_i$ is a predictor of index $CPUE_1$. As we adjusted two models (SBM and BMWSC) to each simulated data set, it was obtained for each index two different estimates: $CPUE_{1\,SBMl}$, $CPUE_{2\,SBMl}$, $CPUE_{3\,SBMl}$ and $CPUE_{1\,BMWSCl}$, $CPUE_{2\,BMWSCl}$, $CPUE_{3\,BMWSCl}$, in which the subscript SBM and BMWSC indicate the model in which the data were interpolated and $l = 1, \ldots, 50$, represents the lth simulation.

The measure used to compare the indices estimates was the mean square error (MSE). Denoting the values of $CPUE_1$, $CPUE_2$, $CPUE_3$ of the lth set of simulated data were as $CPUE_{1l}$, $CPUE_{2l}$, $CPUE_{3l}$, $l = 1, \ldots, 50$; the MSEs are given by:

$MSE_{SBMj} = (1/50)\sum_{l=1}^{50}(CPUE_{jSBMl} - CPUE_{jl})^2$, $j = 1, 2, 3$, for the obtained estimates after the fitting of SBM model.

$MSE_{BMWSCj} = (1/50)\sum_{l=1}^{50}(CPUE_{jBMWSCl} - CPUE_{jl})^2$, $j = 1, 2, 3$, for the obtained estimates after the fitting of BMWSC model. The method with lowest MSEs is considered the best. Just the corresponding estimates for the same index were compared, that is MSE_{SBMj} was confronted with MSE_{BMWSCj} for the same j.

To compare the obtained estimates after the SBM model fitting of the three indices $CPUE_1$, $CPUE_2$ and $CPUE_3$, among each other, the mean absolute relative deviation (MARD) was used, given by

$$MARD_j = \frac{1}{50}\sum_{l=1}^{50}\frac{|CPUE_{jSBM_l} - CPUE_{j_l}|}{CPUE_{j_l}}, j = 1,\ 2,\ 3 \tag{9}$$

where $CPUE_{jSBM_l}$ is the estimate of index $CPUE_{j_l}$ in simulation l and $CPUE_{j_l}$ the true value of the index in simulation l. Each portion of the sum given in Eq. (9) represents the deviation of the estimate from the true value of the index that is being estimated. This criterion is

Table 2
Mean absolute relative deviation (MARD) from the estimates using SBM model from $CPUE_1$, $CPUE_2$ and $CPUE_3$ for each scenario.

Scenario	$CPUE_1$	$CPUE_2$	$CPUE_3$
(a)	0.1161	0.1355	0.6928
(b)	0.0808	0.0791	0.1268
(c)	0.0796	0.0783	0.1689
(d)	0.0492	0.0606	0.1088

therefore not affected by the possible scale difference between the indices.

3. Results and discussion

This section discusses the results of the simulation study. However, before drawing conclusions, the convergence of the posterior distributions obtained by the MCMC methods must be checked. The convergence of the posterior chain of the SBM and BMWSC parameters models were verified by a graphical analysis of their trace. Indications of convergence were obtained when, starting from a certain number of interactions of the algorithm MCMC, the traces of the two chains, which were generated from different initial values, overlapped and started to oscillate around a constant. It was also verified, the capacity of the estimation process in locating the parameters of the SBM model, used in the simulation (Pereira, 2009).

Table 2 shows the results of the mean absolute relative deviation (MARD) of the estimates considering the SBM model, a criterion given by Eq. (9). These results allow a comparison in the estimation of the three abundance indices $CPUE_1$, $CPUE_2$ and $CPUE_3$, considering SBM model.

In all scenarios $CPUE_3$ was the estimated index with the highest MARD values. If a researcher chooses to work with $CPUE_3$ as the CPUE index, the values will not be as well estimated as by $CPUE_1$ or $CPUE_2$ if they are chosen for any of the scenarios (a), (b), (c) or (d).

Comparing $CPUE_1$ with $CPUE_2$, the MARD values by $CPUE_1$ are lower than $CPUE_2$ in (a) and (d). In (b) and (c) the MARD values by $CPUE_2$ are lower, although the values are very close to those by $CPUE_1$. If one wants to use one of the two indices $CPUE_1$ or $CPUE_2$ by the SARD criterion, it is advisable to use $CPUE_1$ for the scenarios (a) and (d) and $CPUE_2$ for (b) and (c), although the loss is not great if $CPUE_1$ is used for these two scenarios also.

In simulation studies conducted by Petrere et al. (2007), the authors stated that the performance of the indices ($CPUE_1$, $CPUE_2$ and $CPUE_3$) depends on the variance and not on the error distribution. These authors did however not analyze the indices in the presence of spatial correlation. Here no association was observed between the covariance structure (the scenarios) and the indices $CPUE_1$, $CPUE_2$ and $CPUE_3$. In general the mean absolute relative deviation (MARD) was lowest for $CPUE_1$ followed by $CPUE_2$.

In Fig. 5 some histograms of estimates are individually presented, considering model SBM in scenario (d), together with their respective medians. This figure illustrates the fact that the estimation method proposed here establishes a sample of estimates for each of the indices, i.e., the histograms give a description of the associated uncertainty. Besides, summaries of interest, such as of credibility intervals can be obtained. An analysis of the his-

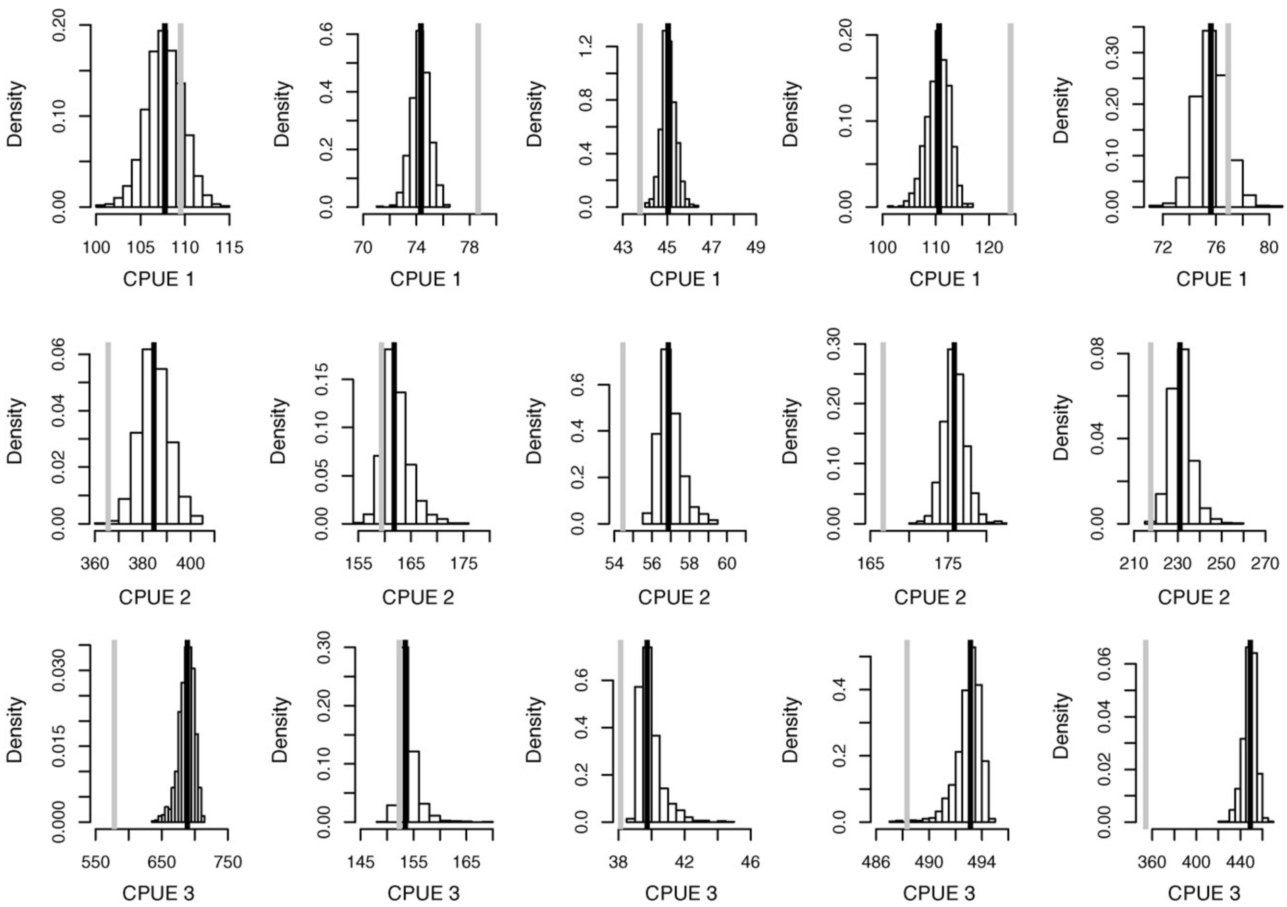

Fig. 5. Histograms of estimates, considering model SBM, corresponding to 5 of 50 samples of $CPUE_1$, $CPUE_2$, $CPUE_3$, with the respective median (in black) and true value (in gray). Scenario (d).

Table 3
Mean square error of adjusted estimates by the models SBM and BMWSC of the indices $CPUE_1$, $CPUE_2$ and $CPUE_3$ for each scenario.

		Scenario			
		(a)	(b)	(c)	(d)
$CPUE_1$	SBM	35.1504	473.4891	5.2285	242.7253
	BMWSC	35.4609	475.3316	7.7926	397.1175
$CPUE_2$	SBM	2.4261	2095.0472	1.8794	2070.7258
	BMWSC	2.4108	2068.0036	2.7029	2962.4262
$CPUE_3$	SBM	2.0064	11038.7357	5.6303	18994.0142
	BMWSC	2.0086	11079.3396	6.6656	19993.4331

tograms of adjusted index estimates, shows that their amplitudes are relatively small. However, all 1000 estimates that compose each histogram are calculated by the data sets consisting of 85 pairs of observed catch and effort and 15 predicted pairs. The 85 pairs of observed catch and effort are common to all estimates, only the 15 predicted pairs change from one estimate to the other (at each iteration of MCMC). That is, all estimates that compose each histogram, have a part in common. This means that the estimates are not very distant from each other, resulting in histograms with small amplitude.

The MSEs resulting from estimates using the models SBM and BMWSC in a simulation study are presented in Table 3.

A comparison of the estimates after adjusting the models SBM and BMWSC shows that the MSE of the adjusted estimate by model BMWSC is greater than the MSE of the adjusted estimate using model SBM for CPUE1 (scenario (a)). The opposite is true for the estimates of CPUE2 and CPUE3. However, the MSE values of the estimates obtained after adjusting model BMWSC for this scenario were similar to the MSE values of the adjusted estimates by model SBM for the same scenario. The results obtained were as expected. In scenario a) there is practically no spatial correlation, so it was expected that the neighboring quadrats would contribute very little to explain what happens in the unobserved quadrats. Furthermore, the correlation catch – effort is low, that is, one variable contributes very little to explain the other (the information available for extrapolation consists in practically the covariate only). It was not expected an expressive gain when using the SBM model in relation to the BMWSC in the estimate of the indexes, once the space correlation is very low for this scenery.

In scenario (b) the spatial correlation *i* low, but the correlation between catch and effort *i* strong. The two models take into account the correlation between the variables, for that it was expected to be little differences in the estimates obtained after the adjustment of the two models. That is, it was expected that the MSE of the estimates obtained after the adjustment of the two models were similar. And it was not expected that one of the models was better than the other for this scenario, the in fact occurred, that is, the SBM model just presented a smaller EQM than the BMWSC model just for $CPUE_1$ and $CPUE_3$.

In scenario (c) on the other hand, the correlation between catch and effort is low, but the spatial correlation is strong. For this scenario the MSEs of the adjusted estimates considering model BMWSC are higher than the MSEs from the adjusted model SBM (Table 3). This result had been expected, because model BMWSC does not consider the spatial correlation observed in this scenario.

For scenario (d), the MSEs of the estimates obtained after adjusting model SBM were always lower (Table 3). In this scenario the spatial correlation is strong, so a great contribution of neighboring quadrats was expected to explain what happens in unobserved quadrats. A considerable gain was expected by the extrapolation to unobserved quadrats using model SBM. In fact, when model SBM is used, there is more information to predict catch and effort at unobserved locations than by model BMWSC. In addition to the correlation between catch and effort, model SBM takes the spatial correlation and the relationship between effort at observed and catch at unobserved locations into account.

Finally, model SBM was better than model BMWSC, where catch and effort are considered spatially independent in the estimation by $CPUE_1$, $CPUE_2$ and $CPUE_3$ for the scenarios (c) and (d) with spatial correlation, which is neglected when model BMWSC is adjusted.

Nishida and Chen (2004) verified, with real data of the yellowfin tuna, that the model in which the spatial correlation is considered it produced a better adjustment to the data giving more realists parameters' estimates. In the present article, we also verified, through the simulation study, that the bivariate model with spatial components, gives better estimates CPUE indices, in those sceneries with spatial correlation in the data.

The main difference of our approach is that with SBM, before estimating the CPUE indices, we modelled effort and capture jointly, so that the crossed spatial covariances are modelled, what is not modelled when it is used other spatial models for the ratio

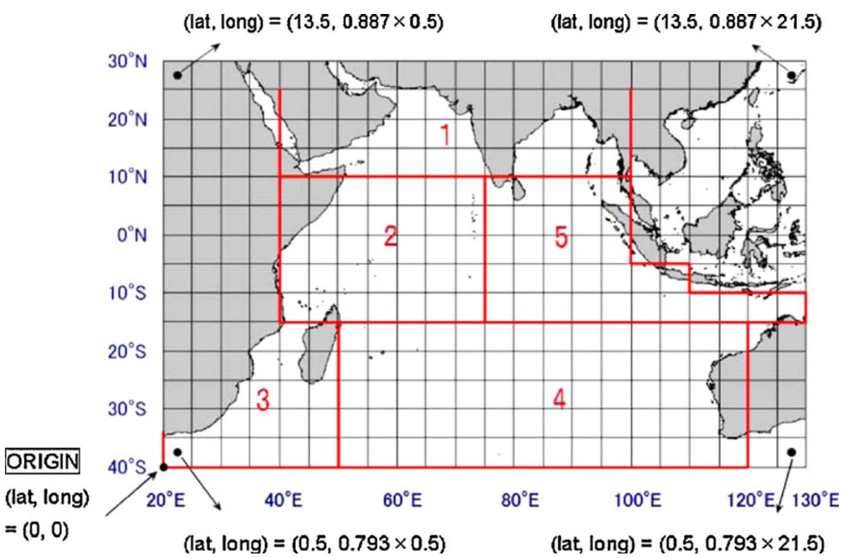

Fig. 6. Definition of the coordinate system for computing the distances between two 5° × 5° areas (the distance of 5° latitudes on the Equator is set to 1). Five subareas adopted by the IOTC (2002) for standardizing yellowfin tuna longline CPUE data in the Indian Ocean (figure from Nishida and Chen (2004)).

between catch and effort as, for instance, spatial-GLM (Nishida and Chen, 2004). On the other hand, when using spatial-GLM it is not necessary to suppose normality, we assume any distribution we judge to be the most appropriate to our data.

As well as in other space approaches, using SBM, it is also possible to do the interpolation (kriging) for no sampled places. Following the Bayesian approach, kriging is carried out using the predictive distribution. Differently from other approaches as (Nishida and Chen (2004) and Stelzenmüller et al. (2007)), that model the ratio between capture and effort, after the adjustment of the SBM model interpolations are calculated catch and effort, being taken into account the correlation between them, the autocorrelation and the crossed covariance. If one wants to get a map of the catch and effort ratio, while other models carry out a kriging for this ratio after the adjustment of SBM a kriging for the pair (E, C) is carried out and it can be obtained, then, any function of those variates, even the ratio catch/effort, with the advantage that more information is used for the kriging.

3.1. Application

An application of the theory was made using data of the Japanese longline yellowfin tuna (Thunnus albacores) fishing in the Indian Ocean. The data refer to the year of 2001. The data set composed by the variates fishing effort, expressed in number of hooks, and the capture in number of individuals per year for a $5^\circ \times 5^\circ$ quadrat. A total of 118 pair of fishing effort and capture data was considered. For each pair there is a reference point in the quadrat where the fishery took place. The subareas adopted by the IOTC Working Party Tropical of Tuna (WPTT) (IOTC, 2002) were also incorporated in the data. Subareas 1, 2, 3, 4 and 5 indicated in Fig. 6 were used as covariates, for ecological reasons (habitat). It is expected that the subareas will help to explain the variates effort and capture. Fig. 6 presents the studied area divided in quadrats of $5^\circ \times 5^\circ$ where the fisheries took place. A system of coordinates was fastened in the area in study in that the point (20thE, 40thS) it was treated as origin. It was defined in this system of coordinates that 5th of latitude on the line of Ecuador represents a unit of distance. In this way, coordinates (x, y) were obtained for the central points of each quadratim (more details can be found in Nishida and Chen (2004)).

The point (20°E, 40°S) was taken as the origin in the study area, in a fixed system where 5° of latitude over the Equator represents the distance unit. So the (x, y) coordinates in the center of each quadrat were calculated according to Nishida and Chen (2004).

A descriptive analysis of the data showed that the statistical distributions of the variates effort and capture are quite asymmetrical and data were taken in log scale. The log(capture) × log (effort) presented a linear correlation of $r = 0.7$. It was observed that the capture and effort data here presented spatial dependency. In this way, the data set is in agreement with scenario (d) of the simulation study, in which the variates are strongly correlated and also with spatial correlation.

The data were adjusted to the models used in the simulation study (SBM and BMWSC), considering as covariate the subareas A_i, $i = 1, 2, 3, 4$ and 5, shown in Fig. 6. The models were adjusted accordingly to the Bayesian approach, as in the simulation study. As a comparison criteria of fitting it was adopted the Deviance Information Criterion (DIC) (Spiegelhalter et al., 2002). Smaller values of DIC indicate a better fit. In Table 4 the DIC values are presented for each fitted model, where we see that the better model is SBM.

Table 4
Deviance Information Criterion (DIC) for each of the models.

Model	DIC
SBM	614.785
BMWSC	765.280

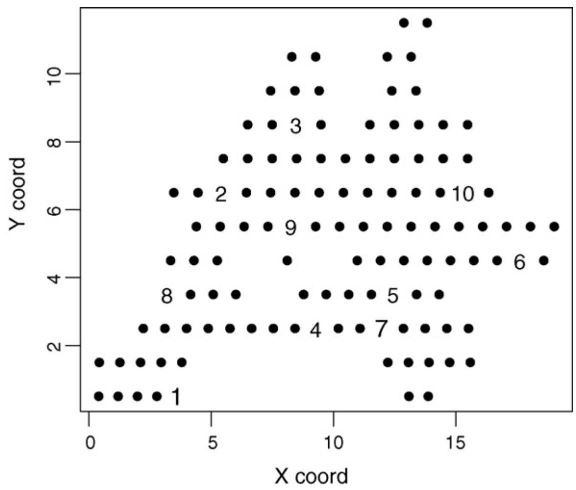

Fig. 7. Reference points of the quadrats where occurred fisheries of yellowfin tuna in 2001. The numbered points were left apart from the inference process in order of being predicted.

During model fitting from the 118 data set, 10 pairs were left out of the inference process in order to evaluate the predictive capacity of the models. In Fig. 7 the enumerated points represent the quadrat centers where effort and catch were kept apart in order to assess their predicted values.

After fitting the models utilizing the predictive distribution estimates of effort and catch, the estimated effort and capture data from the left out observations were calculated. For assessing the model with higher prediction capacity the MSE of each model was calculated. $MSE = \frac{1}{10}\sum_{i=1}^{10}(\hat{Y}_1(s_{ui}) - Y_1(s_{ui}))^2$ and $MSE = \frac{1}{10}\sum_{i=1}^{10}(\hat{Y}_2(s_{ui}) - Y_2(s_{ui}))^2$
in which $Y_1(s_{ui})$ represents the logarithm of the ith location left for prediction and $\hat{Y}_1(s_{ui})$ is its respective predicted value, and Y_2 represents the logarithm of the capture. In Table 5 are shown the MSE values for the two adjusted models. Smaller values indicate better predictions. In Table 5 it is seen that SBM model presented the smaller MSE values for the logarithm of the fishing effort and the logarithm of the captures.

For this data set model SBM presented the best fit and the best predictive capacity of those variates for no observed sites. These results are in agreement with the simulation study. Once the data present spatial correlation and strong correlation between the variates, SBM model seems to be the most appropriate. Consequently, point estimates of the indexes $CPUE_1$, $CPUE_2$ and $CPUE_3$ obtained after the adjustment of SBM model and credibility intervals are more realistic.

In Table 6 the values of the three obtained indices are presented for the complete data set tuna fishery. The estimates obtained after the adjustment of the models SBM and BMWSC are also presented. It is verified that model SBM presents point estimates closer to the indices values and that, while the interval of credibility 95% con-

Table 5
Mean square error (MSE) of the predicted values of $Y_1 = \log(\text{effort})$ and $Y_2 = \log(\text{capture})$.

Model	Variate	
	Y_1	Y_2
SBM	0.4389	0.7334
BMWSC	1.1951	4.8701

Table 6
Value of each index and its respective estimates considering models SBM and BMWSC.

Index	Index value	SBM model		BMWSC model	
		Point estimate	95% credibility interval	Point estimate	95% credibility interval %
$CPUE_1$	0.00534	0.00517	(0.00512, 0.00524)	0,00508	(0.00497, 0.00526)
$CPUE_2$	0.00483	0.00476	(0.00467, 0.00491)	0,00462	(0.00454, 0.00474)
$CPUE_3$	0.00294	0.00298	(0.00289, 0.00319)	0,00287	(0.00285, 0.00290)

sidering model SBM contains the values of the indices $CPUE_2$ and $CPUE_3$, only the interval for $CPUE_1$, considering the model BMWSC contain the value of the index.

3.2. Final observations

The simulation study showed that of the three compared abundance indices, relative deviations were generally lowest for $CPUE_1$ estimates in the scenarios analyzed.

A comparison between models SBM and BMWSC was performed in terms of index estimation. It was found that the estimates of $CPUE_1$, $CPUE_2$ and $CPUE_3$, obtained after adjustment of model SBM, are better than the estimates obtained after adjusting the model BMWSC, for the scenarios (c) and (d) where there is presence of spatial correlation.

If an index of CPUE is applied that does not have ideal properties and/or, if the utilized CPUE is erroneously estimated, wrong decisions may be taken for the fish stock management. Consequently, this study shows that $CPUE_1$, in general is the most appropriate index in the different scenarios discussed and that the model SBM give better estimates of CPUE in some scenarios and can therefore help to avoid mistakes when utilized in fish stock management.

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Appendix A. Complete conditional *a posterior* distributions for the parameters of Eq. (1) model

1. $\boldsymbol{\beta}_1|\phi_1, \sigma_1^2, \mathbf{y}_1 \sim N(\boldsymbol{Bb}, \boldsymbol{B})$, where $\boldsymbol{B} = [\boldsymbol{X}_1^T(\sigma_1^2\boldsymbol{R}_1)^{-1}\boldsymbol{X}_1 + \Sigma_{\boldsymbol{\beta}_1}^{-1}]^{-1}$ and $\boldsymbol{b} = \boldsymbol{X}_1^T(\sigma_1^2\boldsymbol{R}_1)^{-1}\mathbf{y}_1 + \boldsymbol{\Sigma}_{\boldsymbol{\beta}_1}^{-1}\boldsymbol{\mu}_{\beta_1}$;
2. $\sigma_1^2|\boldsymbol{\beta}_1, \phi_1, \mathbf{y}_1 \sim IG\left(\frac{n}{2} + a_{\sigma_1}, c_{\sigma_1}\right)$, where $c_{\sigma_1} = \frac{1}{2}[\mathbf{y}_1 - \boldsymbol{X}_1\boldsymbol{\beta}_1]^T\boldsymbol{R}_1^{-1}[\mathbf{y}_1 - \boldsymbol{X}_1\boldsymbol{\beta}_1]$;
3. $\pi(\phi_1|\boldsymbol{\beta}_1, \sigma_1^2, \mathbf{y}_1) = |\boldsymbol{R}_1|^{-1/2}\exp\left\{-\frac{1}{2}[\mathbf{y}_1 - \boldsymbol{X}_1\boldsymbol{\beta}_1]^T[\sigma_1^2\boldsymbol{R}_1]^{-1}[\mathbf{y}_1 - \boldsymbol{X}_1\boldsymbol{\beta}_1] - b_{\phi_1}\phi_1\right\}\phi_1^{a_{\phi_1}-1}$;
4. $\boldsymbol{\beta}_2|\sigma_2^2, \tau_2^2, \phi_2, y_1, y_2 \sim N(\boldsymbol{B}_2\boldsymbol{b}_2, \boldsymbol{B}_2)$, where $\boldsymbol{B}_2 = [X_2^T(\sigma_2^2\boldsymbol{R}_2 + \tau_2^2\boldsymbol{I})^{-1}\boldsymbol{X}_2 + \boldsymbol{\Sigma}_{\beta_2}^{-1}]^{-1}$ and $\boldsymbol{b}_2 = \boldsymbol{X}_2^T(\sigma_2^2\boldsymbol{R}_2 + \tau_2^2\boldsymbol{I})^{-1}\mathbf{y}_2 + \boldsymbol{\Sigma}_{\beta_2}^{-1}\boldsymbol{\mu}_{\beta_2}$;
5. $\pi(\phi_2|\boldsymbol{\beta}_2, \sigma_2^2, \tau_2^2, \mathbf{y}_1, \mathbf{y}_2) \propto |\sigma_2^2\boldsymbol{R}_2 + \tau_2^2\boldsymbol{I}|^{-1/2}\exp\left\{-\frac{1}{2}[\mathbf{y}_2 - \boldsymbol{X}_2\boldsymbol{\beta}_2]^T[\sigma_2^2\boldsymbol{R}_2 + \tau_2^2\boldsymbol{I}]^{-1}[\mathbf{y}_2 - \boldsymbol{X}_2\boldsymbol{\beta}_2] - b_{\phi_2}\phi_2\right\}\phi_2^{a_{\phi_2}-1}$;
6. $\pi(\sigma_2^2|\boldsymbol{\beta}_2, \tau_2^2, \phi_2, \mathbf{y}_1, \mathbf{y}_2) \propto |\sigma_2^2\boldsymbol{R}_2 + \tau_2^2\boldsymbol{I}|^{-1/2}\exp\left\{-\frac{1}{2}[\mathbf{y}_2 - \boldsymbol{X}_2\boldsymbol{\beta}_2]^T[\sigma_2^2\boldsymbol{R}_2 + \tau_2^2\boldsymbol{I}]^{-1}[\mathbf{y}_2 - \boldsymbol{X}_2\boldsymbol{\beta}_2] - \frac{b_{\sigma_2}}{\sigma_2^2}\right\}(\sigma_2^2)^{-a_{\sigma_2}-1}$;
7. $\pi(\tau_2^2|\boldsymbol{\beta}_2, \sigma_2^2, \phi_2, \mathbf{y}_1, \mathbf{y}_2) \propto |\sigma_2^2\boldsymbol{R}_2 + \tau_2^2\boldsymbol{I}|^{-1/2}\exp\left\{-\frac{1}{2}[\mathbf{y}_2 - \boldsymbol{X}_2\boldsymbol{\beta}_2]^T[\sigma_2^2\boldsymbol{R}_2 + \tau_2^2\boldsymbol{I}]^{-1}[\mathbf{y}_2 - \boldsymbol{X}_2\boldsymbol{\beta}_2] - \frac{b_{\tau_2}}{\tau_2^2}\right\}(\tau_2^2)^{-a_{\tau_2}-1}$.

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